$1/N$ expansion: methods and applications

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Outline

- QFT and critical phenomena
- 1/N expansion:
  self-consistency equations, conformal bootstrap, etc.
- MS like scheme:
  technical details and recent results

- Systems at the phase transition point.

Observables are critical exponents: $\langle A(x)A(0)\rangle \sim |x|^{-\Delta}$

Universality: $\{\Delta\}$ depend only on the characteristics of the critical point.

Different physical systems have the same critical behavior

- K. Wilson, 1973 : ϵ expansion

QFT in $d = 4 - 2\epsilon$ dimensions can be used to describe phase transitions.

The coupling constant depends on the scale

$$M\frac{dg}{dM} = \beta(g,\epsilon) \qquad \beta(g_*(\epsilon),\epsilon) = 0$$

$g_*(\epsilon)$ is a critical coupling, $g_* \sim \epsilon \mapsto$ perturbative methods

The critical exponents $\Delta(\epsilon) = \gamma(g_*(\epsilon),\epsilon)$.

$\Delta(\epsilon) = \Delta_0 + \Delta_1\epsilon + \Delta_2\epsilon^2 + \ldots \quad \epsilon \mapsto 1/2.$

Huge progress in the area of multi-loop calculations:

- QCD 5-loops
  Baikov, Chetyrkin, Kühn, Vermaseren, Herzog, Ruijl, . . .
- φ^4 6-loops
  Kompaniets, Panzer, Batkovich, Chetyrkin, ...

There are alternative approaches to calculation of critical indices:

- Numerical methods
- Analytic methods for low dimensional systems $/2d$ CFT, **Polyakov, Zamolodchikov,....**
- Conformal bootstrap methods / **Showk, Paulos, Poland, Rychkov, Simmons-Duffin,...**
- $1/N$ expansion

N component φ^4 model vs nonlinear σ model: $\varphi^2 = \sum_{i=1}^{N} \varphi_i^2$

$$S_{\varphi^4}(\varphi) = \int d^D x \left(\frac{1}{2}(\partial\varphi)^2 + \frac{g}{2N}(\varphi^2)^2 \right)$$

critical at

$$u_* = \frac{g_*}{16\pi^2} = \frac{6\epsilon}{N+8} + O(\epsilon).$$

$$S_{\varphi^4}(\varphi) \mapsto S(\varphi, \sigma) = \int d^D x \left(\frac{1}{2}(\partial\varphi)^2 + \sigma((\varphi)^2) - \frac{N}{2g}\sigma^2 \right)$$

σ field propagator: $(2\mu = D)$

$$D_\sigma^{-1}(p) \sim N(p^2)^{\mu-2}\left(1 + p^{2\epsilon}/g\right) \underset{p\to 0}{\sim} N(p^2)^{\mu-2}$$

Both theories are in the same universality class.

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One can compute the critical indices in $1/N$ expansion in D dimensions:

$$\Delta(D) = \Delta_0(D) + \frac{1}{N}\Delta_1(D) + \frac{1}{N^2}\Delta_2(D) + \ldots$$

The functions $\Delta_k(D)$ contain information on contributions ϵ^p/N^k in perturbative expansion, p is arbitrary.

RG functions in the MS scheme do not depend on dimension D

$$\gamma(g) = M\frac{d}{dM}\ln Z(\epsilon, g) = \sum\nolimits_k \gamma_k(N) g^k$$

At the critical point

$$\gamma(g) \mapsto \gamma(g_*) = \sum\nolimits_k \gamma_k(N) g_*^k(\epsilon) \qquad\qquad g_*(\epsilon) \sim \epsilon/N + \ldots$$

Critical dimension $\Delta(D) = \Delta_{\text{can}} + \gamma(g_*)$.

Comparing MS and $1/N$

$$\Delta(D) = \sum_k \gamma_k(N) g_*^k(\epsilon) \qquad (MS)$$

$$\Delta(D) = \sum_k 1/N^k \Delta_k(\epsilon) \qquad (1/N)$$

The expansion coefficients $\gamma_k(N)$ are polynomials in N:

$$\begin{aligned}\gamma_k(N) &= \gamma_{kk} N^k + \gamma_{kk-1} N^{k-1} + \ldots + \gamma_{k1} N + \gamma_{k0} \\ &= N^k \left(\gamma_{kk} + \gamma_{kk-1}/N + \ldots \gamma_{k0}/N^k\right)\end{aligned}$$

Additional check of perturbative calculations.

Vasil'ev, Pis'mak, Honkonen, 83, γ_φ at $1/N^3$ in σ model

Vasil'ev, Derkachov, Kivel, Stepanenko, 93; Gracey 94, γ_q at $1/N^3$ in GN model,

Interpolation between $2 + \epsilon$ and $4 - \epsilon$.

Methods

- Self-consistency equations

  [**Schwinger-Dyson equations with dressed propagators**]

  Vasil'ev, Pis'mak, Honkonen, 83, γ_φ at $1/N^2$.

  J. Gracey, Application to different models (including gauge theories)

- Conformal bootstrap

  [**Schwinger-Dyson equations with dressed propagators and vertices**]

  Vasil'ev, Pis'mak, Honkonen, 83, γ_φ at $1/N^3$ in σ model,

  Vasil'ev, Derkachov, Kivel, Stepanenko, 93; Gracey 94, γ_q at $1/N^3$ in GN model,

- "MS scheme"

  Vasil'ev, Nalimov, 83, Analog of dimensional regularization: $D_\sigma(p) = 1/p^{2(\mu-2+\Delta)}$.

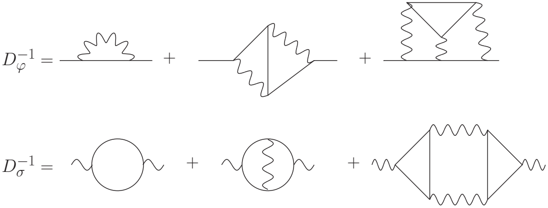
  Vasil'ev, Stepanenko, 93; Derkachov, A.M, 97

  Up to $1/N^2$ order the critical indices can be calculated via Z factors.

Self-consistency equations

Schwinger-Dyson equations with dressed propagators:

$$D_\varphi(x) = \frac{A}{x^{2\alpha}} \qquad\qquad D_\sigma(x) = \frac{B}{x^{2\beta}}$$



$$A^{-1} p(\alpha) x^{-2(D-\alpha)} + AB x^{-2(\alpha+\beta)} = 0 \qquad B^{-1} p(\beta) x^{-2(D-\beta)} + N A^2 x^{-4\alpha} = 0$$

where

$$p(\alpha) = \pi^{-D} \frac{\Gamma(\alpha)\Gamma(D-\alpha)}{\Gamma(D/2-\alpha)\Gamma(\alpha-D/2)}$$

Self-consistency equations

$$p(\alpha) = \pi^{-D} \frac{\Gamma(\alpha)\Gamma(D-\alpha)}{\Gamma(D/2-\alpha)\Gamma(\alpha-D/2)}$$

The amplitudes A, B can be excluded and one gets equation on indices:

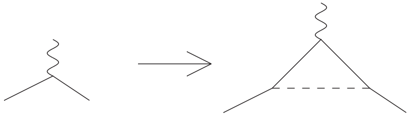
$$p(\alpha) + u = 0 \qquad 2p(\beta)/N + u = 0 \qquad u = A^2 B \qquad 2\alpha + \beta = D \equiv 2\mu$$

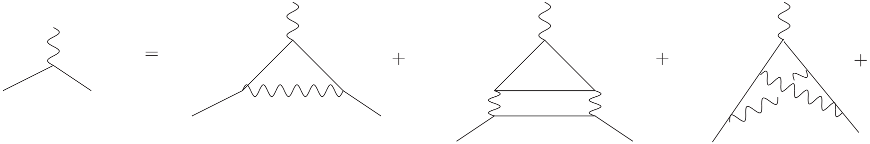
$$p(\alpha) = 2p(\beta)/N$$

$\alpha = \mu - 1 + \eta/2$ and $\beta = 2 - \eta$ $\quad p(\alpha) \sim \eta \cdots$

$$\eta = -\frac{4}{N} \frac{\Gamma(2\mu-2)}{\Gamma(2-\mu)\Gamma(\mu-2)\Gamma(\mu-1)\Gamma(\mu+1)}$$

Schwinger-Dyson equations with dressed propagators and vertices: Structure of a three point correlator is fixed by the conformal symmetry





$+$ six more diagrams (to determine η with $1/N^3$ accuracy).

Vasil'ev, Pis'mak, Honkonen, 83, index η at $1/N^3$ in σ model,

Vasil'ev, Derkachov, Kivel, Stepanenko, 93; Gracey 94, index η at $1/N^3$ in GN model,

Vasil'ev, Stepanenko, 93, index $1/\nu$ at $1/N^2$ in GN model,

Pismensky, 2015, index η at ϵ^4 in φ^3 model,

J. Gracey,

Conformal Methods for Massless Feynman Integrals and Large Nf Methods, 17
Large Nf quantum field theory, 13

Vasil'ev, Nalimov, 83

$$S(\varphi,\sigma) = \int Dx\Big(\frac{1}{2}(\partial\varphi)^2 + \sigma\varphi^2\Big) \mapsto$$
$$= \int Dx\Big(Z_1\frac{1}{2}(\partial\varphi)^2 + u\frac{1}{2}\sigma(x)\int Dy K_\Delta(x-y)\sigma(y) + Z_2\sigma\varphi^2 - v\frac{1}{2}\sigma(x)\int Dy K(x-y)\sigma(y)\Big)$$

$K_\Delta(x) = A/(x^2)^{D-2}(M^2x^2)^\Delta$. K cancels the simple loop insertion of φ field in the propagator of σ field.

The model is renormalizable, but not multiplicatively renormalizable,

$$S_R(\varphi,\sigma) \neq S(\varphi_0,\sigma_0)$$

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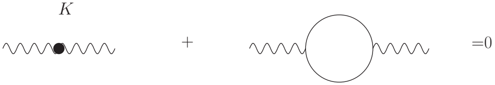

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Extended model: $S(\varphi,\sigma) \mapsto S(\varphi,\sigma,u,v) \qquad S(\varphi,\sigma) = S(\varphi,\sigma,u=1,v=1)$

Divergencies appears as poles in Δ

RG equations

$$\Big(M\partial_M + \beta_u\partial_u + \beta_v\partial_v + \gamma_\Gamma\Big)\Gamma(u,v,p_i) = 0$$

$\beta_u = M\frac{du}{dM}$, $\beta_v = M\frac{dv}{dM}$.

$$\beta_u = \beta_v = \gamma_\sigma \neq 0$$

σ field propagator

$$D_\sigma(x) = B/(x^2)^{2-\Delta} \Longrightarrow D_\sigma(x) = \frac{1}{u} B/(x^2)^{2-\Delta} \sum_{m=0}^{\infty} \left(\tfrac{v-1}{u}\right)^m (x^2)^{-m\Delta}$$
$$= \frac{1}{u} B/(x^2)^{2-\Delta} \left(1 + \left(\frac{v-1}{u}\right) x^{-2\Delta} + \dots\right)$$

RG equation contains $\Gamma(u,v)$ and the derivative $(\partial_u + \partial_v)\Gamma(u,v)$.

$$D_\sigma^{\Delta=0}(x) = \frac{1}{u}\frac{B}{x^4} \sum_{m=0}^{\infty} \left(\tfrac{v-1}{u}\right)^m = \tfrac{B}{x^4}\tfrac{1}{1+u-v}$$

In $1/N$ expansion:

$$\Gamma(u,v) = \Gamma_0(u-v) + \frac{1}{N}\Gamma_1(u,v) + \dots$$

Vasil'ev, Nalimov, 83: $\Gamma(u,v) = \Gamma(u-v)$ in the MOM scheme.

$\Big(M\partial_M + \gamma_\Gamma\Big)\Gamma(u-v,p_i) = 0 \qquad \gamma_\Gamma$ is the scaling dimension of the correlator.

In any other scheme

$$\Gamma(u,v,p) = \widetilde{Z}(u,v)\Gamma(u-v,p) \qquad\qquad \beta_u(\partial_u+\partial_v)\Gamma(u,v,p) = \Delta\gamma_\Gamma\Gamma(u,v,p)$$

$\Delta\gamma = \beta_u(\partial_u + \partial_v)\log\widetilde{Z}$.

Minimal subtraction scheme: $Z = \sum_n Z_n/\Delta^n$**:** $\qquad \Gamma(u,v) \neq \Gamma(u-v)$

$$\gamma_\Gamma \neq \tilde{\gamma}_\Gamma = M\frac{d\log Z}{dM}$$

S. Derkachov, A.M., 98

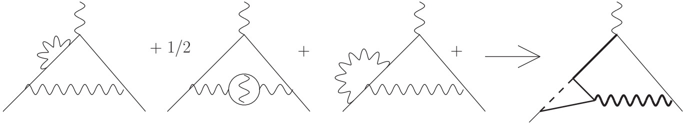
MS scheme

$$\Gamma(u,v) = \Gamma_0(u-v) + \frac{1}{N}\Gamma_1(u-v) + \frac{1}{N^2}\Gamma_2(u,v) + \ldots$$

$$\gamma_\Gamma - \tilde{\gamma}_\Gamma = O(1/N^3)\,.$$

$$\tilde{\gamma}_\Gamma = -2\sum_{\Gamma_i} n_{\sigma,i} \times Z_{i,1} (= \text{simple pole residue})$$

The self-energy and vertex correction diagrams resummation :



The dressed propagators and vertices have the form

$$D_\varphi(x) = \widehat{A}/x^{2\Delta_\varphi}\,, \qquad D_\sigma(x) = \widehat{B}/x^{2\Delta_\sigma}\,,$$

and

$$\gamma_R(z,x,y) \equiv \Gamma_{\sigma\varphi\varphi}(z,x,y) = \widehat{Z}(z-x)^{-2\alpha}(z-y)^{-2\alpha}(x-y)^{-2\beta}\,.$$

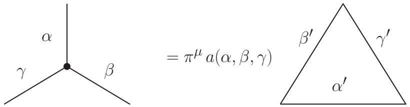
Here

$$\Delta_\varphi = \mu - 1 + \gamma_\varphi\,, \qquad \Delta_\sigma = 2 + \gamma_\sigma\,,$$
$$\alpha = \mu - 1 - \gamma_\sigma/2\,, \qquad \beta = 2 - \gamma_\varphi + \gamma_\sigma/2\,.$$

Uniqueness condition:

$$\alpha + \beta + \Delta_\varphi = D \qquad 2\alpha + \Delta_\sigma = D$$

Star-triangle relation $\alpha + \beta + \gamma = D \equiv 2\mu$.



$$\alpha' = \mu - \alpha, \quad a(\alpha, \beta, \gamma) = a(\alpha)a(\beta)a(\gamma), \; a(\alpha) = \Gamma(\mu - \alpha)/\Gamma(\alpha).$$

The diagram with the dressed vertices and propagators has only a surface divergence:

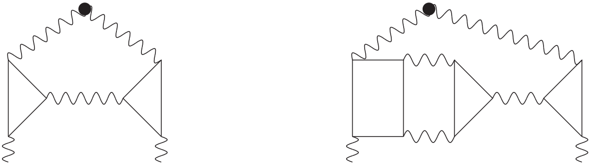
$$D = \frac{R}{\Delta} + F$$

$$\delta\gamma_{SE+V} = \delta\gamma_1/N + \delta\gamma_2/N^2 + \ldots = -2R$$

This trick reduces the number of diagrams and greatly simplifies calculations.

MS-like scheme

Example: σ^2 operator: $$D(\Delta) = \frac{1}{\Delta} R + \ldots$$



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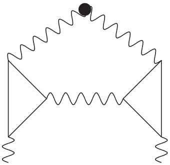
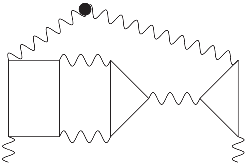
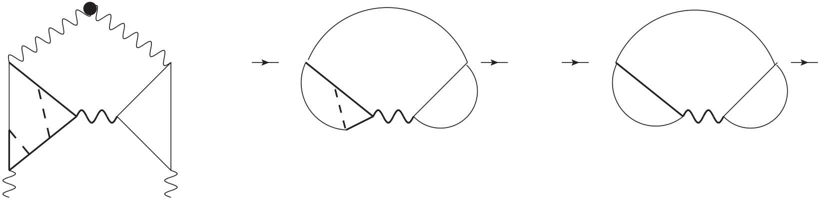






- Anomalous dimensions via Z-factors
- Dressed propagators and vertices ($+$ star-triangle relation)

- Critical dimensions of σ^ℓ and non-singlet twist two operators $\varphi(\otimes\partial)^n\varphi$ in the nonlinear σ model (NSM), **Derkachov, A.M, 98**
- γ_m in QCD at $1/N_f^2$, **Ciuchini, Derkachov, Gracey, A.M, 99**
- Twist two singlet operators $\bar{q}(\otimes\partial)^n q$ in the Gross-Neveu model **Skvortsov, A.M, 16**
- Twist two singlet operators $\varphi(\otimes\partial)^n\varphi$ in the NSM **Skvortsov, Strohmaier, A.M, 17**
- σ^3 and $\sigma\partial^2\sigma$ in the GN model, **Strohmaier, A.M, 17**

  Zerf, Mihaila, Marquard, Herbut, Scherer, 17
  Four-loop critical exponents for the Gross-Neveu-Yukawa models:
  two couplings, g_1, g_2, $\quad \beta_k(g_q, g_2)$, $k = 1, 2$, $\quad \omega_{ik} = \partial_{g_i}\beta_k(g_1, g_2)$.

Klebanov, Polyakov, 2002 the vector $O(N)$ model is dual to HST AdS$_4$. (Conjecture)
Leigh, Petkou, 2003 GN model

$$J_s \equiv J_{\mu_1\ldots\mu_s} = \varphi\partial_{\mu_1}\ldots\partial_{\mu_s}\varphi + \text{total derivatives} \qquad (NLS)$$

$$J_s \equiv J_{\mu_1\ldots\mu_s} = \bar{q}\gamma_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_s}q + \text{total derivatives} \qquad (GN)$$

$$m_s^2 = m_0^2(s) + \delta m_s^2\,, \qquad m_0^2(s) = (d+s-2)(s-2) - s, \qquad \delta m_s^2 = \gamma_s(d-4+2s+\gamma_s)\,.$$

Muta and Popovic, 77 (GN-model)

$$\gamma_{ns}(s) = \frac{1}{n}\eta_{GN}\left(1 - \frac{\mu(\mu-1)}{j_s(j_s-1)}\right)\,,$$

$$\gamma(s) = \frac{1}{n}\eta_{GN}\left(1 - \frac{\mu(\mu-1)}{j_s(j_s-1)}\left(1 + \frac{\Gamma(2\mu-1)}{\mu-1}\frac{\Gamma(j_s-\mu+2)}{\Gamma(j_s+\mu-2)}\right)\right)$$

$j_s = s + \mu - 1.$

Lang and Ruhl, 99 (NLS-model) In the leading order in $1/n$ (up to $\eta_{GN} \to \eta_{NSM}$)

$$\gamma_{NSM}(s) = \gamma_{GN}(s)$$

Gribov Lipatov reciprocity relaion – Large j asymptotic of anomalous dimensions
Dokshitzer, Marchesini, Salam, 06, Basso, Korchemsky, 07

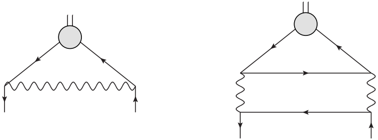
$$\gamma(s) = f\left(j_s + \frac{1}{2}\gamma_s\right) \qquad\qquad f(j) \sim \left(j - \frac{1}{2}\right)^{-\Delta_q} \sum_{k\geq 0} \frac{a_{q,k}}{(j(j-1))^k}\,.$$

$$\gamma(s) = f_1(j_s) + \frac{1}{2} f_1(j_s) f_1'(j_s) + f_2(j_s) + \ldots$$

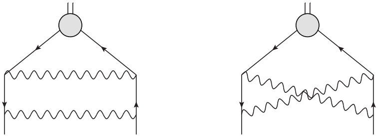
Alday, Zhiboedov, 17

Proof based on: conformal bootstrap + crossing symmetry + unitarity

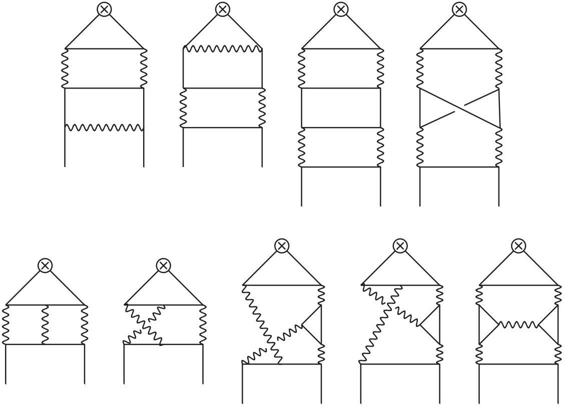
LO diagrams:



NLO non-singlet diagrams



NLO singlet diagrams:



Any diagram without divergent subgraphs has the “reciprocity" property.

$$\gamma(s) = f(j) = \eta\Big(1 + f_1(j) + \Delta f_1(j)\Big) + \eta^2\Big(f_2^+(j) + \Delta f_2(j)\Big) + O(1/n^3)\,,$$

where $j = s + \mu - 1 + \gamma(s)/2$

$$\Delta f_1(j) = f_1(j)\,\frac{\Gamma(2\mu-1)}{\mu-1}\,\frac{\Gamma(j-\mu+2)}{\Gamma(j+\mu-2)}\,.$$

For the function $\Delta f_2(j)$ we obtained

$$\begin{aligned}\Delta f_2(j) = \Delta f_1(j)\Bigg\{ & -\Big[\psi(j)-\psi(\mu)\Big] - \frac{2\mu-1}{\mu-1}\Big(\Psi(j,\mu)-\Psi(\mu,\mu)\Big) - \frac{1}{2}\Big(\psi(2-\mu)-\psi(\mu)\Big) \\ & - \frac{1}{2}\,\frac{1}{j(j-1)} + \frac{(2\mu-3)(3\mu-1)}{2(\mu-1)(j-\mu+1)(j+\mu-2)} + \frac{1}{(\mu-1)^2} + \frac{1}{2\mu(\mu-1)} - 1 \\ & - \frac{1}{2}f_1(j)\Bigg(\psi(1-\mu)-\psi(\mu-1)-2+\frac{2\mu-3}{(j-\mu+1)(j+\mu-2)}\Bigg) \\ & - \frac{1}{2}\Delta f_1(j)\Bigg(\psi(2-\mu)-\psi(\mu)-1+\frac{1}{(j-\mu+1)(j+\mu-2)} \\ & - \frac{j(j-1)}{(\mu-1)(j-\mu+1)(j-2+\mu)}\Big[\Psi(j,\mu)+\psi(\mu)-\psi(1)-\Upsilon(j,\mu)\Big]\Bigg)\Bigg\}\,,\end{aligned}$$

where

$$\Psi(j,\mu)=\psi(j-2+\mu)+\psi(j+2-\mu)-2\psi(j)$$

and

$$\Upsilon(j,\mu)=\frac{\Gamma(j)}{\Gamma(\mu-2)\Gamma(j+2-\mu)(j+\mu-2)}\int_0^1 du\, u^{\mu-2}\bar{u}^{j-\mu}\,{}_2F_1\left(\begin{matrix}1,1\\ j+\mu-1\end{matrix}\Big|-\frac{u}{\bar{u}}\right)$$

$$=\frac{1}{\Gamma(\mu-2)}\frac{\Gamma(j-2+\mu)}{\Gamma(j+2-\mu)}\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dt\,\Gamma^2(t+1)\Gamma(\mu+t)\Gamma(-t)\frac{\Gamma(j-\mu+1-t)}{\Gamma(j+\mu-1+t)}\,.$$

$$\gamma(s) = \frac{\eta_1}{N}\gamma_1(s) + \left(\frac{\eta_1}{N}\right)^2 \gamma_2(s) + \ldots$$

In $D = 3 \quad \eta_1 = \frac{8}{3\pi^2}$ **and**

$$\gamma_1(s) = \frac{2(s-2)}{2s-1}\,,$$

$$\gamma_2(s) = \frac{3}{4s^2-1}\Bigg(-\frac{32s^2}{9} - \frac{\left(13s^2+14s+6\right)\log(2)}{s} - \frac{3}{2}\pi s + \frac{3}{2}s\left(S_1\left(\frac{s}{2}+\frac{3}{4}\right) - S_1\left(\frac{s}{2}+\frac{1}{4}\right)\right)$$
$$+ \frac{3(-1-s+s^2)}{s}\left(S_1\left(\frac{s}{2}\right) - S_1\left(\frac{s+1}{2}\right) - S_1(s+1)\right) - \frac{(s+2)(7s+6)}{2s}S_1\left(s+\frac{1}{2}\right)$$
$$+ \frac{\left(13s^2+3s+3\right)}{s}S_1(s) + 13s - \frac{9}{s+1} + \frac{1}{2s-1} - \frac{6}{(2s-1)^2} - \frac{3}{2s+1} + \frac{9}{2s+3} - \frac{9}{s} + \frac{152}{9}\Bigg).$$

$$\delta m_s^2 = 2\eta(s-2)\Big(1 + \eta\varkappa(s) + \dots\Big).$$

At large spin $\varkappa(s) = \frac{39}{8}\frac{\log s}{s} + \dots$

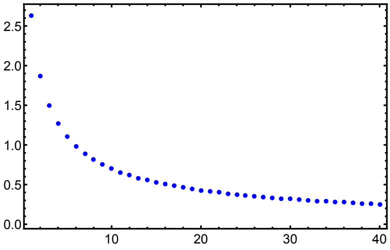


Figure: Function $\varkappa(2k)$.

Conclusions

- $1/N$ methods provide a nontrivial check of perturbative calculations.
- "MS like scheme" is very effective up to $1/N^2$ order.