Statistical physics of polymerized phantom membranes

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Workshop on multi-loop calculations – Methods and Applications

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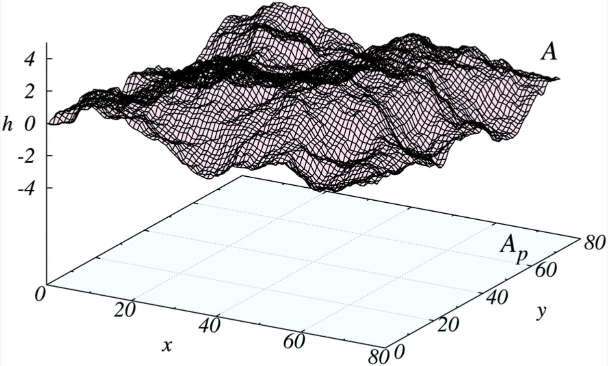
Outline

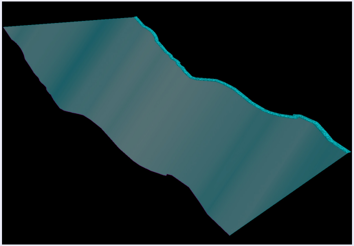
Introduction

- membranes: D-dimensional extended objects embedded in a d-dimensional space subject to quantum and/or thermal fluctuations



- High energy physics: (sum over) surfaces occurs within, e.g.:
  - strong coupling expansion of lattice gauge theory
  - discretization of Euclidean quantum gravity

- string theory (Polyakov, David, Foerster (70's - 80's))
  - a string sweeps out a surface (worldsheet)



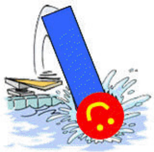
(see also branes . . . (Polchinski 90's, . . .))

- chemical physics / biology :

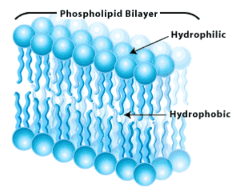
(Nelson - Peliti, Helfrich, , Aronovitz - Lubensky, David - Guitter, Le Doussal - Radzihovsky, . . . (70's - 90's))

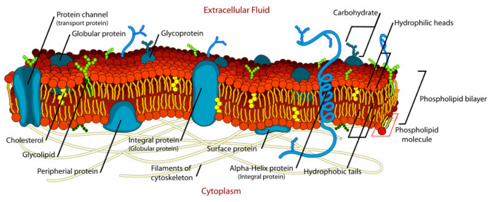
$\Longrightarrow$ structures made of amphiphile molecules (ex: phospholipid)

- one hydrophilic head
- hydrophobic tails



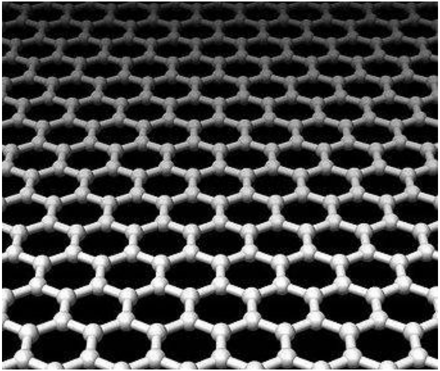
$\Longrightarrow$ bilayers $\Longrightarrow$ living cell





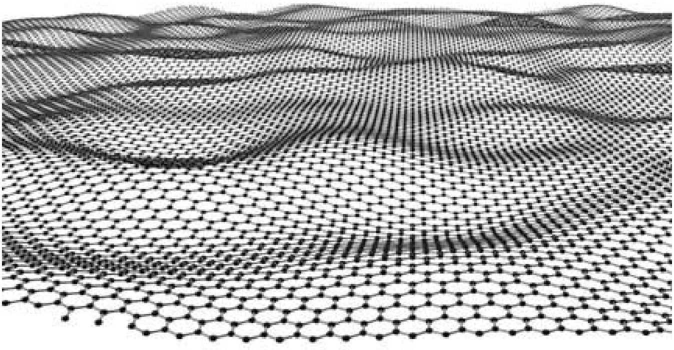
- condensed matter physics: graphene, silicene, phosphorene . . .

  uni-layers of atoms located on a honeycomb lattice
- striking properties:
  - high electronic mobility, transmittance, conductivity,. . .



- mechanical properties: both extremely strong and soft material:

  $\Longrightarrow$ example of genuine 2D fluctuating membrane

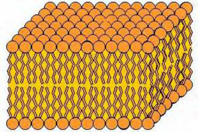


Generic questions :

- effects of – thermal – fluctuations ?
- does a flat – ferromagnetic-like – phase exist at low temperatures ?

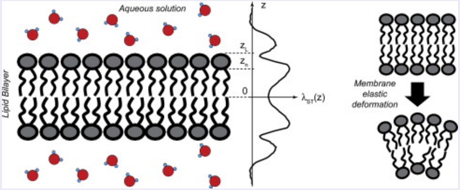
Fluid membranes *vs* polymerized membranes

Fluid membranes



- weakly interacting molecules
  - free diffusion inside the membrane plane $\Longrightarrow$ vanishing shear modulus
  - very small compressibility and elasticity $\Longrightarrow$ no elastic energy

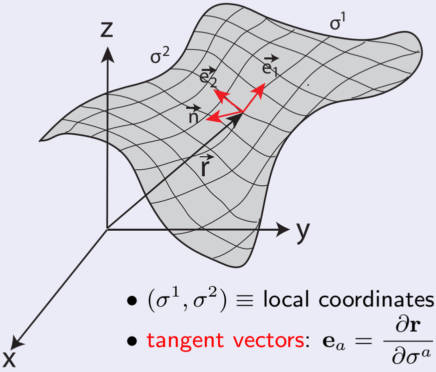
$\Longrightarrow$ main contribution to the energy: *bending energy*



Free energy of fluid membranes

- point of the surface described by the embedding:

$\mathbf{r}$: $\boldsymbol{\sigma} = (\sigma^1, \sigma^2) \to \mathbf{r}(\sigma^1, \sigma^2) \in \mathbb{R}^d$



- $(\sigma^1, \sigma^2) \equiv$ local coordinates on the membrane
- tangent vectors: $\mathbf{e}_a = \dfrac{\partial \mathbf{r}}{\partial \sigma^a} = \partial_a \mathbf{r} \qquad a = 1, 2$
- unit-norm normal vector : $\hat{\mathbf{n}} = \dfrac{\mathbf{e}_1 \times \mathbf{e}_2}{|\mathbf{e}_1 \times \mathbf{e}_2|}$

- curvature tensor $\mathbf{K}$: $K_{ab} = -\hat{\mathbf{n}}.\,\partial_b \mathbf{e}_a = \mathbf{e}_a.\,\partial_b \hat{\mathbf{n}}$
- $K_{ab} \Longrightarrow$ locally diagonalized with eigenvalues K_1 and K_2
  - mean or *extrinsic* curvature:

$$H = \frac{1}{2}(K_1 + K_2) = \frac{1}{2}\mathrm{Tr}\,\mathbf{K}$$

  - Gaussian or *intrinsic* curvature: $K = K_1\,K_2 = \det K_a{}^b$

    $\Longrightarrow$ no role with a fixed topology (Gauss-Bonnet theorem)

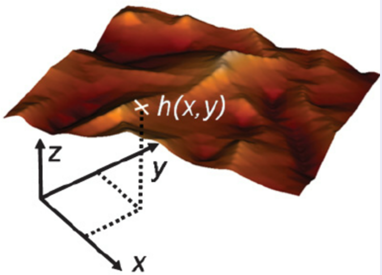
$\Longrightarrow$ bending energy:

$$F = \frac{\kappa}{2}\int d^2\boldsymbol{\sigma}\,\sqrt{g}\;H^2$$

- κ: rigidity constant
- $g_{\mu\nu} = \partial_\mu \mathbf{r}.\partial_\nu \mathbf{r} \equiv$ metric induced by the embedding $\mathbf{r}(\boldsymbol{\sigma})$
- $\sqrt{g} = \sqrt{\det g_{\mu\nu}}$ ensures the reparametrization invariance

Fluctuations ?

- Low temperatures: Monge parametrization
  $x = \sigma_1$, $y = \sigma_2$ and $z = h(x,y)$ with h height, capillary, mode



- $\mathbf{r}(x,y) = (x, y, h(x,y))$
- $\hat{\mathbf{n}}(x,y) = \dfrac{(-\partial_x h, -\partial_y h, 1)}{\sqrt{1+|\boldsymbol{\nabla} h|^2}}$
- $\hat{\mathbf{n}}(x,y).\,\mathbf{e_z} = \cos\theta(x,y) = \dfrac{1}{\sqrt{1+|\boldsymbol{\nabla} h|^2}}$

- Free energy:

$$F \simeq \frac{\kappa}{2} \int d^2\mathbf{x}\ (\Delta h)^2 + \mathcal{O}(h^4)$$

- is there a flat phase ? $\Longrightarrow$ fluctuations of $\theta(x,y)$:

$$\langle \theta(x,y)^2 \rangle = k_B T \int d^2 q\ \frac{1}{\kappa\, q^2} \simeq \frac{k_B T}{\kappa} \ln\left(\frac{1}{qa}\right) \underset{q \to 0}{\rightarrow} \infty$$

$\Longrightarrow$ no !

$\Longrightarrow$ no long-range order between the normals

in agreement with Mermin-Wagner theorem

At next order in h, κ is renormalized and decreases at long distances.:

$$\kappa_R(q) = \kappa - \frac{3k_BT}{2\pi}\left(\frac{d}{2}\right)\ \ln\left(\frac{1}{qa}\right)$$

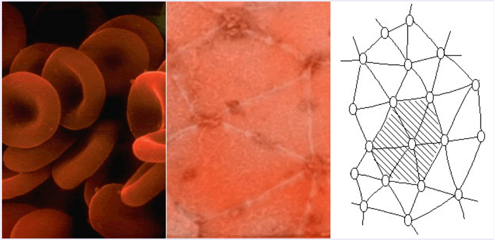
$\Longrightarrow$ divergence of $\langle\theta(x,y)^2\rangle$: worse

$\Longrightarrow$ strong analogy with 2D-nonlinear σ model:

- exp. decreasing correlations: $\langle\hat{\mathbf{S}}(\mathbf{r}).\hat{\mathbf{S}}(\mathbf{0})\rangle \sim e^{-r/\xi}$
- correlation length – mass gap: $\xi \simeq a\, e^{2\pi\kappa/3k_BT(d/2)}$
- nothing really new but $N-2 \Longrightarrow d/2$

Polymerized membranes

- chemical physics/biology: red blood cell, ...
- condensed matter physics: graphene, phosphorene, ...



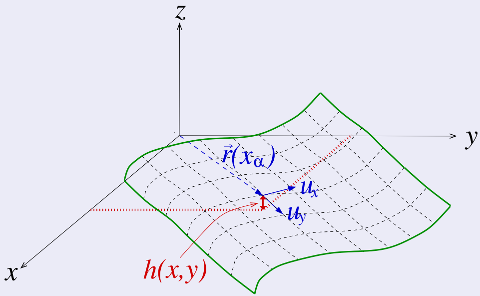
- membranes made of molecules interacting by $V(|\mathbf{r_i} - \mathbf{r_j}|)$

$\Longrightarrow$ bending and elastic energy contributions

Free energy of crystalline membranes

- Flat reference configuration: $\mathbf{r}_0(x, y) = (x, y, z = 0)$
- Fluctuations: $\mathbf{r}(x, y) = \mathbf{r}_0 + u_x(x, y)\,\mathbf{e}_1 + u_y(x, y)\,\mathbf{e}_2 + h(x, y)\,\hat{\mathbf{n}}$

  $h \equiv$ height field and $u_i \equiv$ phonon fields



- Free energy:

$$F \simeq \int d^2\mathbf{x}\ \left[\frac{\kappa}{2}(\Delta h)^2 + \mu (u_{ab})^2 + \frac{\lambda}{2}(u_{ab})^2\right]$$

$u_{ab} \equiv$ stress tensor $\sim$ encodes fluctuations with respect to the flat configuration $\mathbf{r}_0$

$u_{ab} = \frac{1}{2}\left[\partial_a u_b + \partial_b u_a + \partial_a \mathbf{u}.\partial_b \mathbf{u} + \partial_a h\, \partial_b h\right]$

$\kappa \equiv$ rigidity constant $\quad \lambda,\ \mu \equiv$ elastic coupling constants

- coupling between height and phonon fluctuations

$\Longrightarrow$ *frustration* of height fluctuations

First approach of fluctuations

Gaussian approximation on phonon fields:

$$u_{ab} \simeq \frac{1}{2}\left[\partial_a u_b + \partial_b u_a + \partial_a h\, \partial_b h\right]$$

integrate over u:

$$F_{eff} = \frac{\kappa}{2}\int d^2\mathbf{x}\ (\Delta h)^2 + \frac{\mathcal{K}}{8}\int d^2\mathbf{x}\left(P^T_{ab}\,\partial_a h\,\partial_b h\right)^2$$

- $P^T_{ab} = \delta_{ab} - \partial_a\partial_b/\nabla^2$
- κ = bending, rigidity modulus
- $\mathcal{K} = 4\mu(\lambda+\mu)/(2\mu+\lambda)$: Young elasticity modulus

- Self-consistent screening approximation (SCSA) $\sim$ Schwinger-Dyson equation in the large d limit (Nelson and Peliti 87)

$$\kappa_{eff}(\mathbf{q}) = \kappa + k_B T \mathcal{K} \int d^2k \frac{\left[\hat{q}_a\, P^T_{ab}\, \hat{q}_b\right]^2}{\kappa_{eff}(\mathbf{q}+\mathbf{k})|\mathbf{q}+\mathbf{k}|^4}$$

$$\Longrightarrow \kappa_{eff}(\mathbf{q} \to \mathbf{0}) \sim \frac{\sqrt{k_B T \mathcal{K}}}{q}$$

i.e. effective rigidity increased by fluctuations !

- fluctuations of normal vector $\hat{\mathbf{n}}$:

$$\langle \theta(x,y)^2 \rangle = k_B T \int d^2q \, \frac{1}{\kappa_{eff}(\mathbf{q}) q^2} < \infty \, !$$

$\Longrightarrow$ Long-range order between normals in $D=2$ (and less) !

- no trouble with Mermin-Wagner Theorem

F_{eff} can be rewritten as an interaction between Gaussian curvatures:

$$F_{eff} = \frac{\kappa}{2} \int d^2\mathbf{x}\ (\Delta h)^2 + \mathcal{K} \int_{\mathbf{x},\mathbf{y}} K(\mathbf{x}) G(\mathbf{x}-\mathbf{y}) K(\mathbf{y})$$

$K \equiv$ Gaussian curvature:

$$K(\mathbf{x}) = -\Delta(\partial_a h\, \partial_b h) + \partial_a \partial_b (\partial_a h\, \partial_b h)$$

with a non decreasing kernel:

$$G(\mathbf{x}-\mathbf{y}) \propto |\mathbf{x}-\mathbf{y}|^2 \ln |\mathbf{x}-\mathbf{y}|$$

$\Longrightarrow$ evades Mermin-Wagner Theorem !

- polymerized membranes: possibility of spontaneous symmetry breaking in $D = 2$ and even in $D < 2$

  $\Longrightarrow$ low-temperature, ordered, flat, phase with non-trivial correlations in the I.R.

$$\begin{cases} G_{hh}(\mathbf{q}) \sim q^{-(4-\eta)} \\ \\ G_{uu}(\mathbf{q}) \sim q^{-(6-D-2\eta)} \end{cases}$$

  with $\eta \neq 0 \Longrightarrow$ associated *e.g.* to stable sheet of graphene

- a challenge: to compute η

- which also provides the lower critical dimension: $D_{lc}(\eta) = 2 - \eta$

Renormalization group approaches to polymerized membranes

One-loop perturbative approach of the flat phase

(Aronovitz, Golubović and Lubensky 88; Guitter, David, Leibler, Peliti 89)

$$F \simeq \int d^D\mathbf{x} \ \left[\frac{\kappa}{2}(\Delta h)^2 + \mu(u_{ab})^2 + \frac{\lambda}{2}(u_{aa})^2\right]$$

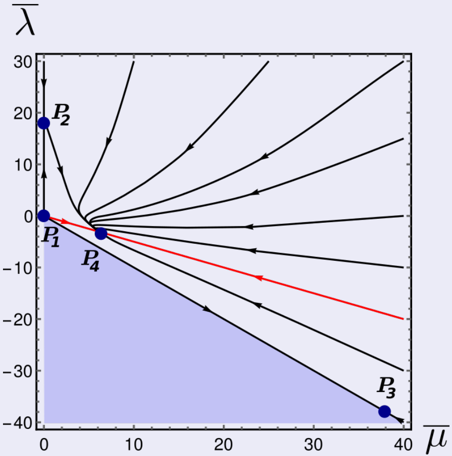
$\Longrightarrow$ perturbative expansion in λ and μ

β-functions in $D = 4 - \epsilon$ at one-loop order:

$$\begin{aligned}\partial_t \mu &= (-\epsilon + 2\eta)\mu + \frac{d_c \mu^2}{96\pi^2} \\ \partial_t \lambda &= (-\epsilon + 2\eta)\lambda + \frac{d_c(6\lambda^2 + 6\lambda\mu + \mu^2)}{96\pi^2}\end{aligned}$$

with $d_c = d - D$ (nb of Goldstone modes) and $\eta = 5\mu(\lambda+\mu)/(2\mu+\lambda)$

- 3 unstable fixed points P_1, P_2 and P_3
- a non-trivial fixed point P_4 that governs the flat phase and is located on the submanifold $3\lambda + \mu = 0$



Physical properties

- a nonvanishing value of $\eta = \dfrac{12\epsilon}{d - D + 24}$
  with: $\eta(\epsilon = 2, d = 3) = 0.96$
- increased rigidity: $\kappa_{eff}(L) \sim L^{\eta}$
- decreased elasticity modulus: $\mathcal{K}_{eff}(L) \sim L^{-2+2\eta}$

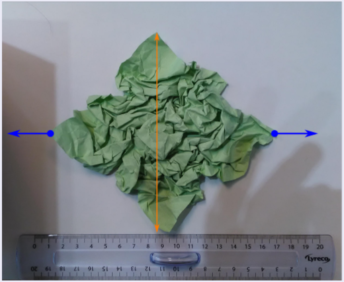
$\Longrightarrow$ consequences of "ripples"

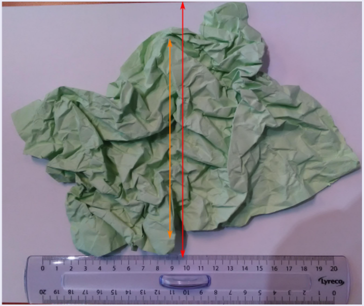
Physical property

- Poisson ratio : $\nu = -\dfrac{\text{expansion}_T}{\text{expansion}_L}$

- as the flow runs towards $3\lambda + \mu = 0$: $\nu = \dfrac{\lambda^*}{3\lambda^* + 2\mu^*} = -\dfrac{1}{3}$

  in agreement with experiments





Everything seems to be fine . . .

However:

- flat phase properties (η and ν) poorly determined in $D = 2$ – far from $D = 4$ – and $d = 3$
- SCSA or weak-coupling expansion extremely tedious beyond leading order due to :
  - multiplicity of fields: h, u and the coupling constants λ, μ
  - derivative nature/complexity of the interaction

$\Longrightarrow$ use of a non perturbative RG approach
(J.-P. Kownacki and D. Mouhanna 08)

Non perturbative renormalization group

Wilson program: (K.G. Wilson, L.P. Kadanoff, J. B. Kogut ...(70's))

- gradual integration over high momentum fluctuations

$$\mathcal{Z} = \int \mathcal{D}\zeta \; e^{-\mathrm{H}[\zeta]}$$

splitting: $\zeta(\boldsymbol{q}) = \zeta_>(\boldsymbol{q}) + \zeta_<(\boldsymbol{q})$

with: $$\begin{cases} \zeta_>(\boldsymbol{q}) = \zeta(\boldsymbol{q}) & \text{for} \quad k \leq q \leq \Lambda = a^{-1} \\ \zeta_<(\boldsymbol{q}) = \zeta(\boldsymbol{q}) & \text{for} \quad 0 \leq q \leq k \end{cases}$$

$$\mathcal{Z} = \int \mathcal{D}\zeta_< \; e^{-\mathrm{H}_k[\zeta_<]}$$

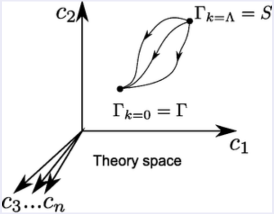
Wilson-Polchinski equation:

$$\partial_k \mathrm{H}_k[\zeta_<] = \frac{1}{2} \int_q \partial_k C_>(\mathbf{q}) . \left(\frac{\delta^2 \mathrm{H}_k}{\delta\zeta_<(\mathbf{q})\delta\zeta_<(-\mathbf{q})} - \frac{\delta \mathrm{H}_k}{\delta\zeta_<(\mathbf{q})} \frac{\delta \mathrm{H}_k}{\delta\zeta_<(-\mathbf{q})} \right)$$

- formulation in terms of Hamiltonian not very efficient . . .

- best formulation in terms of running effective action (or running Gibbs free energy) Γ_k in which high-momentum fluctuations – $k \leq q \leq \Lambda$ – have been integrated out (Wetterich (90's))

- $\Gamma_{k=\Lambda} = H[\zeta] \;\equiv\;$ microscopic scale
- $\Gamma_k \;\; = \Gamma_k[\phi] \;\equiv\;$ running scale k
- $\Gamma_{k=0} = \Gamma[\phi] \;\equiv\;$ macroscopic scale



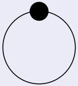
Γ_k follows an exact equation (Wetterich (93)):

$$\boxed{\partial_k \Gamma_k[\phi] = \frac{1}{2}\int d^d\boldsymbol{q}\ \partial_k R_k(\boldsymbol{q}^2)\ \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(\boldsymbol{q}^2)}}$$

with $R_k(\boldsymbol{q}^2) \equiv$ cut-off function

- one-loop structure close to that of perturbative field theory

$$\partial_k \Gamma_k[\phi] = \quad \frac{1}{2}$$



but:

- $R_k(\boldsymbol{q}^2) \underset{q\to 0}{\sim} k^2 \Longrightarrow$ I.R. finiteness
- $R_k(\boldsymbol{q}^2) \underset{q\to \infty}{\to} 0 \Longrightarrow$ U.V. finiteness
- $\Gamma_k^{(2)}[\phi]_{ij} \equiv$ full, *i.e.* field-dependent propagator !

  $\Longrightarrow$ non perturbative in the coupling constants, d, D, etc !

- Effective action $\Gamma_k[\partial_\mu \mathbf{r}]$ for membranes: (Kownacki and Mouhanna 08)

  - ansatz for $\Gamma_k\left[\partial_\mu \mathbf{r}\right]$: bending and elastic terms

$$\Gamma_k\left[\partial_\mu \mathbf{r}\right] = \int d^D\mathbf{x}\ \frac{\kappa}{2}\left(\Delta \mathbf{r}\right)^2 + \ \lambda\left(\partial_a \mathbf{r}.\partial_b \mathbf{r}\right)^2 + \ \mu\left(\partial_a \mathbf{r}.\partial_a \mathbf{r}\right)^2$$

$+$ power-counting non renormalizable terms !

$\Longrightarrow$ compute $\Gamma_k^{(2)}\left[\partial_\mu \mathbf{r}\right]$

$\Longrightarrow$ plug it into the Wetterich's equation

$\Longrightarrow$ compute the (non perturbative) RG equations

$$\partial_t\lambda = (D-4+2\eta_t)\lambda + \frac{256\ \tilde{d}\ \lambda^2\ \tilde{A}_D}{D(D+2)(D+4)(D+8)}$$

$$\partial_t\mu = (D-4+2\eta_t)\mu + \frac{128\ \tilde{d}\ (\lambda^2 + 2(D+2)\lambda\mu + D(D+2)\mu^2)\tilde{A}_D}{D(D+2)(D+4)(D+8)}$$

with the running anomalous dimension

$$\eta_t = \frac{128(D+4)(D^2-1)\lambda(\lambda+2\mu)A_D}{D^2(D^2+6D+8)(\lambda+\mu) + 128(D^2-1)\ \lambda(\lambda+2\mu)A_D}$$

with $\tilde{A}_D = A_D(8+D\eta_t)$ and $A_D = 2^{-D-1}\pi^{-D/2}/\Gamma(D/2)$

$\Longrightarrow$ non polynomial/non perturbative in λ and μ

Results: (Kownacki and Mouhanna 08)

- flow runs towards the hypersurface: $(2+D)\lambda + 2\mu = 0$

$$\Longrightarrow \nu = \frac{\lambda^*}{(D-1)\lambda^* + 2\mu^*} = -\frac{1}{3} \text{ in } any\ D \ !$$

- $\eta = 0.849$ that compares very well to Monte Carlo computation with an interatomic potential for graphene: $\eta = 0.850$! (Los, Katsnelson, Yazyev, Zakharchenko and Fasolino 09)

At higher orders. . . ?

- no corrections at higher orders in $(\partial \mathbf{r})^{2p}$!
  (Essafi, Kownacki and Mouhanna 14)
- no quantitative corrections at all orders in ∂^{2p} !
  $\nu = -1/3$ and $\eta = 0.85$
  (Braghin and Hasselmann 10)

$\Longrightarrow$ extreme – unexpected – stability of this approach

also: high powers in fields and field-derivatives in $\Gamma_k[\partial_\mu \mathbf{r}] \Longleftrightarrow$ high orders of perturbation theory

Question: what is structure/properties of this theory at higher orders in perturbation in λ and μ ?

Polymerized membranes at two-loop order

Polymerized membranes at two-loop order

(Coquand, Mouhanna and Teber, to be published)

$$S[\vec{u},\vec{h}] = \frac{1}{2}\int d^Dx \Big[\kappa(\Delta\vec{h})^2$$
$$+\lambda\left((\partial_i u_i)^2 + \partial_i u_i(\partial_j\vec{h}.\partial_j\vec{h}) + \tfrac{1}{4}(\partial_j\vec{h}.\partial_i\vec{h})^2\right)$$
$$+\mu\left((\partial_i u_i)^2 + \partial_i u_j\partial_j u_j + \partial_i u_j(\partial_i\vec{h}.\partial_j\vec{h}) + i\leftrightarrow j + \tfrac{1}{2}(\partial_j\vec{h}.\partial_i\vec{h})^2\right)\Big]$$

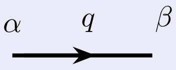
Properties

- derivative field theory $\Longrightarrow$ momentum dependent vertices
- theory "living" in space-time: no internal degrees of freedom h_α with $\alpha = 1\ldots d-D$ and u_i with $i = 1\ldots D$

Feynman rules

height-field propagator:

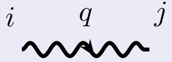
$$G_h^{\alpha\beta}(q) = \frac{\delta^{\alpha\beta}}{q^4} =$$



Phonon-field propagator:

$$G_u^{ij}(q) = \frac{1}{\mu q^2} P_T^{ij}(q) + \frac{1}{(\lambda + 2\mu) q^2} P_L^{ij}(q)$$

$$=$$



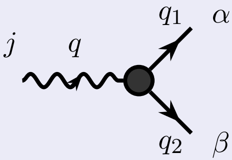
with $P_T^{ij}(q) = \delta_{ij} - \dfrac{q_i q_j}{q^2}$ and $P_L^{ij}(q) = \dfrac{q_i q_j}{q^2}$

Feynman rules

3-points (momentum-dependent) vertex: hhu

$$V^{j}_{\alpha\beta}(q) = -\frac{i}{2}\Big[\lambda(q_1.q_2)q^j + \mu\big((q.q_1)q_2^j + (q.q_2)q_1^j\big)\Big]\delta_{\alpha\beta}$$

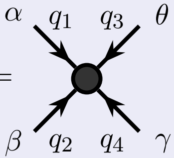
$=$



Feynman rules

4-points (momentum-dependent) vertex: $hhhh$

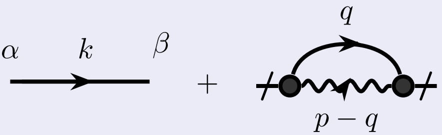
$$
\begin{aligned}
W_{\alpha\beta\gamma\theta}(q) \underset{q_1+q_2=q}{=} \frac{1}{24}\Big\{ & \lambda\Big[(q_1.q_2)(q_3.q_4)\delta_{\alpha\beta}\delta_{\gamma\theta} \\
& + (q_1.q_3)(q_2.q_4)\delta_{\alpha\gamma}\delta_{\beta\theta} \\
& + (q_1.q_4)(q_2.q_3)\delta_{\alpha\theta}\delta_{\beta\gamma}\Big] \\
& + \mu\Big[\big((q_1.q_3)(q_2.q_4) + (q_1.q_4)(q_2.q_3)\big)\delta_{\alpha\beta}\delta_{\gamma\theta} \\
& + \big((q_1.q_4)(q_2.q_3) + (q_1.q_2)(q_3.q_4)\big)\delta_{\alpha\gamma}\delta_{\beta\theta} \\
& + \big((q_1.q_2)(q_3.q_4) + (q_1.q_3)(q_2.q_4)\big)\delta_{\alpha\theta}\delta_{\beta\gamma}\Big]\Big\}
\end{aligned}
$$

$=$ 

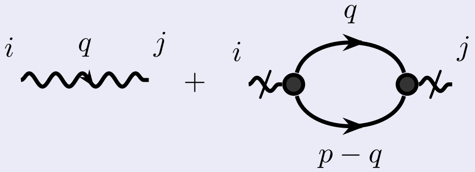
Good news: it is sufficient to renormalize the propagators !

One-loop

$$\left[\Sigma_h(p)\right]_{1-loop} =$$

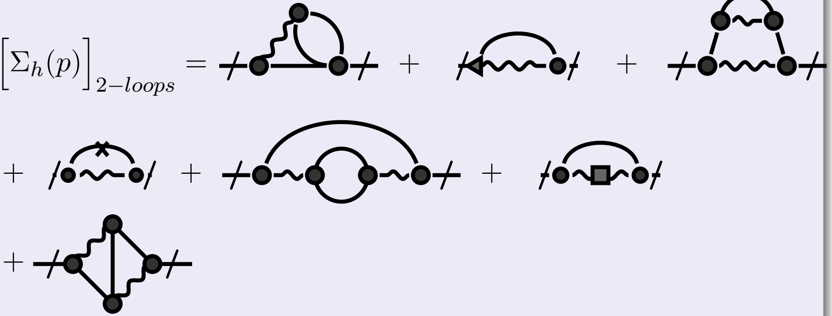


$$\left[\Sigma_u^{ij}(p)\right]_{1-loop} =$$



Two-loop

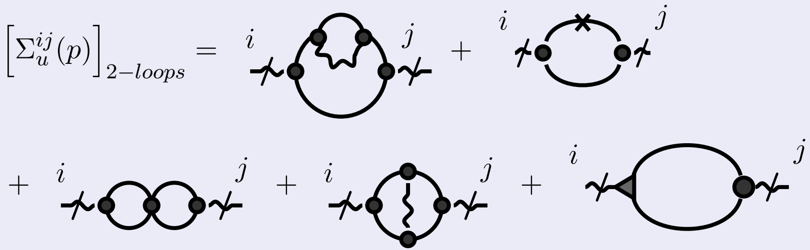
$$\Big[\Sigma_h(p)\Big]_{2-loops} =$$



Involve integrals of the kind – and kite ! :

$$J_2(D, \vec{p}, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = \int \frac{d^D k_1 d^D k_2}{(\vec{p}-\vec{k}_1)^{2\alpha_1}(\vec{p}-\vec{k}_1-\vec{k}_2)^{2\alpha_2}(\vec{p}-\vec{k}_2)^{2\alpha_3} k_1^{2\alpha_4} k_2^{2\alpha_4}}$$

$$\left[\Sigma_u^{ij}(p)\right]_{2-loops} =$$



Results

- A non-trivial stable fixed point P_4 that controls the flat phase with:

$$\lambda^* = -\epsilon \, \frac{32\pi^2}{d_c + 24} + \epsilon^2 \, \frac{32\pi^2(19d_c + 156)}{5(d_c + 20)^3}$$

$$\mu^* = \epsilon \, \frac{96\pi^2}{d_c + 24} - \epsilon^2 \, \frac{32\pi^2(47d_c + 228)}{5(d_c + 24)^3}$$

  but the flow is no longer goes towards the submanifold $3\lambda + \mu = 0$!

Physical properties

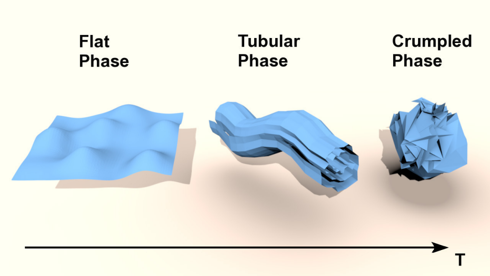
- Poisson ratio:
  one-loop: $\nu = -\frac{1}{3} = -0.3333$ and two-loop: $\nu = -0.2460$!
- anomalous dimension in $D = 2$ (*i.e.* $\epsilon = 2\dots$) and $d = 3$:
  one-loop: $\eta = 0.96$ and two-loop: $\eta = 0.9139$ to be compared to $\eta = 0.85$

Conclusion

- Satisfying qualitative and quantitative description of polymerized membranes by means of non perturbative RG and stability with respect to higher orders
- One exhibits a conflict with perturbation theory
  $\Longrightarrow$ strong dependence with respect to the order

- Observed in other situations:

- Anisotropic membranes $\Longrightarrow$ tubular phase



- anisotropy between the x and y directions

$$\Gamma_k[\mathbf{r}] = \int d^{D-1}x \; dy \left\{ \frac{Z_y}{2}(\partial_y^2 \mathbf{r})^2 + t_x(\partial_x \mathbf{r})^2 + \frac{u_y}{2}(\partial_y \mathbf{r}.\partial_y \mathbf{r})^2 \right\}$$

- transition between a crumpled phase with $\zeta_y = 0$ at high T and a tubular phase with $\zeta_y \neq 0$ at low T

Upper critical dimension: $D = 5/2$ close to $D = 2$

$\Longrightarrow \epsilon = 5/2 - D$ expansion in good position ?

- perturbatively: $\eta = -0.0015 < 0$! (rigidity: $\kappa \sim 1/q^{\eta}$)
  ϵ-expansion: *"unreliable"* and *"qualitatively wrong"*
  (Radzihovsky and Toner 95)

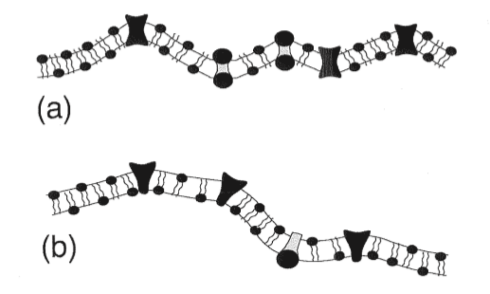
- non perturbatively:
  (Essafi, Kownacki and Mouhanna 11)
  $\eta = 0.358(4) > 0$ in agreement with MC data ...

- Disordered membranes

  (Coquand, Kownacki, Essafi and Mouhanna 18)

  disorder exists: imperfect polymerization, vacancies, proteins,

  $\Longrightarrow$ elastic (a) and curvature (b) disorder



Hamiltonian: $\mathbf{c}(\mathbf{x})$ and $m(x)$ Gaussian random fields

$$H[\mathbf{r}] = \int \mathrm{d}^D x \left\{ \frac{\kappa}{2} \Big(\partial_\mu \partial_\mu \mathbf{r}(\mathbf{x}) - \mathbf{c}(\mathbf{x}) \Big)^2 + \lambda \Big(\partial_\mu \mathbf{r}(\mathbf{x}).\partial_\nu \mathbf{r}(\mathbf{x}) - \delta_{\mu\nu}\, m(x) \Big)^2 + \mu \Big(\partial_\mu \mathbf{r}(\mathbf{x}).\partial_\mu .\mathbf{r}(\mathbf{x}) - D\, m(x) \Big)^2 \right\}$$

$\Longrightarrow$ modelize curvature and elasticity disorder

- quenched disorder $\Longrightarrow$ average over disorder of $\overline{F} = \overline{\log Z}$ using replica trick:

$$\overline{\log Z} = \lim_{n \to 0} \frac{\overline{Z^n - 1}}{n}$$

$\Longrightarrow$ effective action with interacting replica :

$$\Gamma[\mathbf{r}] = \int \mathrm{d}^d x \sum_{\alpha} \left\{ \frac{\overline{\kappa}}{2} \Big(\partial_i \partial_i \mathbf{r}^\alpha(\mathbf{x})\Big)^2 + \frac{\overline{\lambda}}{8} \Big(\partial_i \mathbf{r}^\alpha(\mathbf{x}).\partial_i \mathbf{r}^\alpha(\mathbf{x})\Big)^2 + \frac{\overline{\mu}}{4} \Big(\partial_i \mathbf{r}^\alpha(\mathbf{x}).\partial_j \mathbf{r}^\alpha(\mathbf{x})\Big)^2 \right\}$$
$$- \frac{\overline{\Delta}_\kappa}{2} \sum_{\alpha,\beta} \partial_i \partial_i \mathbf{r}^\alpha(\mathbf{x}).\partial_j \partial_j \mathbf{r}^\beta(\mathbf{x})$$
$$- \frac{\overline{\Delta}_\lambda}{8} \sum_{\alpha,\beta} \Big(\partial_i \mathbf{r}^\alpha(\mathbf{x}).\partial_i \mathbf{r}^\alpha(\mathbf{x})\Big)\Big(\partial_j \mathbf{r}^\beta(\mathbf{x}).\partial_j \mathbf{r}^\beta(\mathbf{x})\Big)$$
$$- \frac{\overline{\Delta}_\mu}{8} \sum_{\alpha,\beta} \Big(\partial_i \mathbf{r}^\alpha(\mathbf{x}).\partial_j \mathbf{r}^\alpha(\mathbf{x})\Big)\Big(\partial_i \mathbf{r}^\beta(\mathbf{x}).\partial_j \mathbf{r}^\beta(\mathbf{x})\Big)$$

with $\overline{\Delta}_\kappa, \overline{\Delta}_\lambda, \overline{\Delta}_\mu$ disorder variances

- non perturbative RG approach : *stable fixed point* (Coquand, Essafi, Kownacki, Mouhanna 18):

  not seen within perturbation theory
- at one-loop order (Morse and Lubensky 92)
- at two-loop order . . . (Coquand, Mouhanna and Teber, to be published)

- there is a clear need for a clarification of the link between perturbative and non perturbative approaches
- require studies of high orders of perturbation theory