Binary Black Holes and Gluon Scattering Amplitudes

Mikhail P. Solon

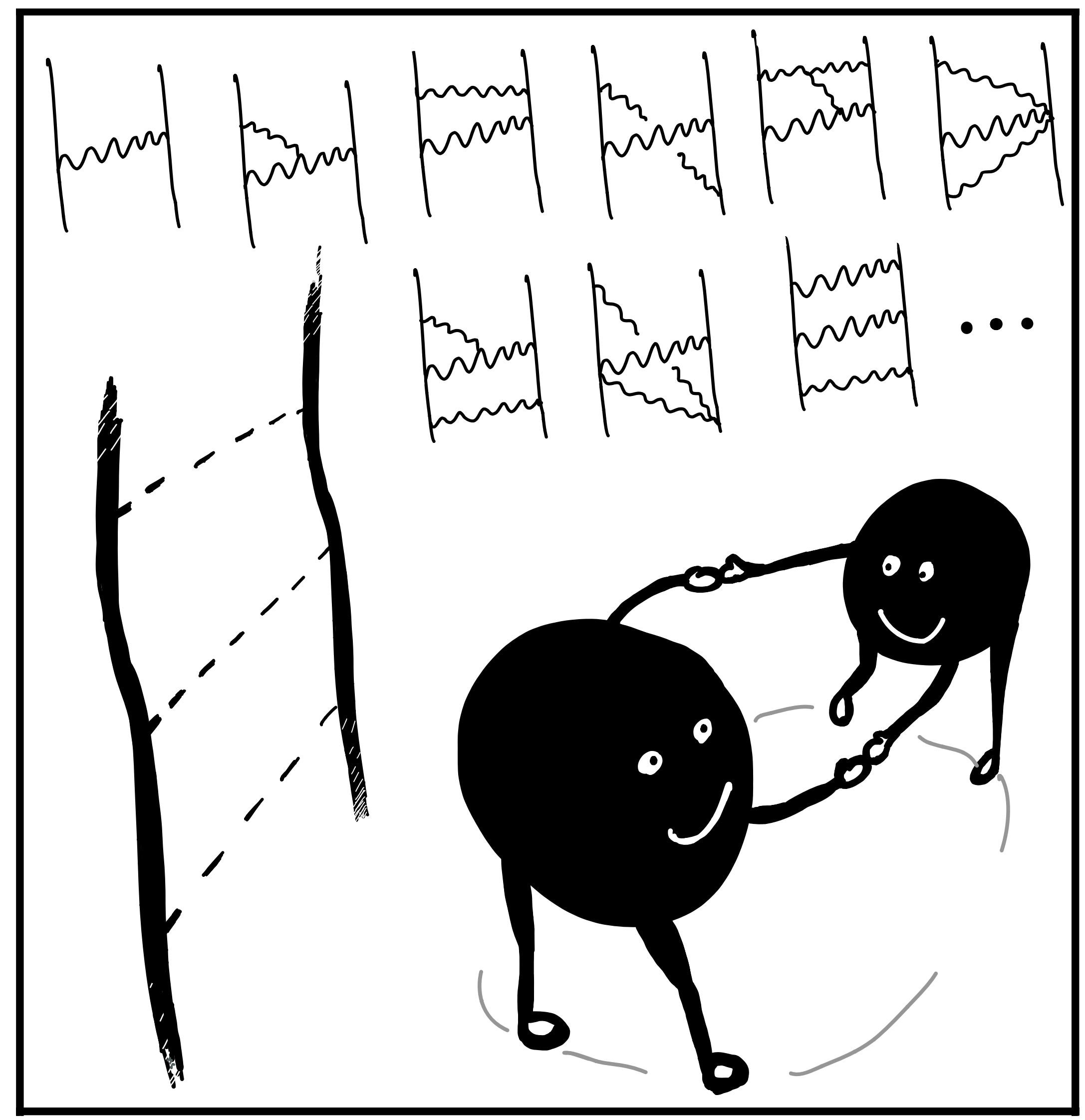
Caltech

based on work with

Clifford Cheung, Ira Rothstein (PRL)

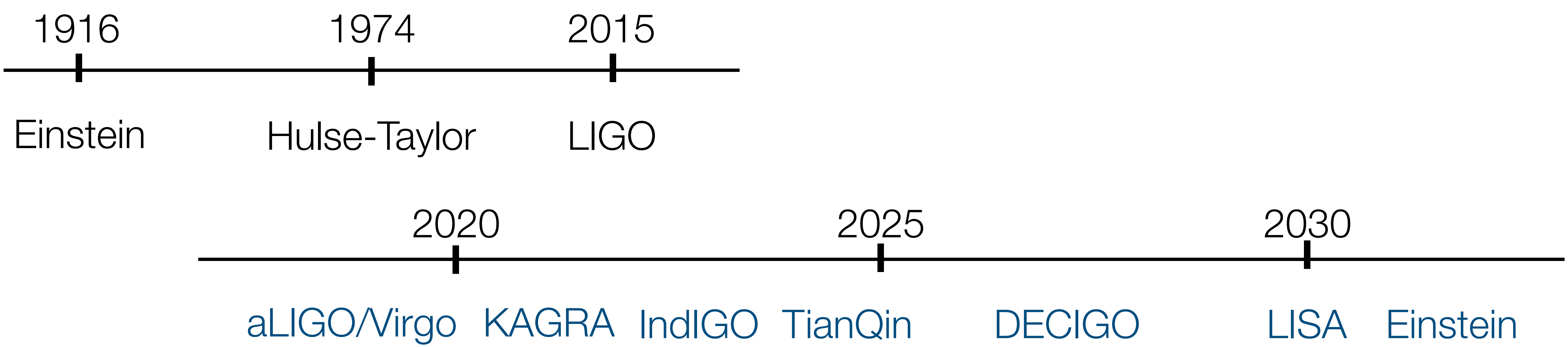
Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mao Zeng (PRL)

Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mao Zeng (long paper)



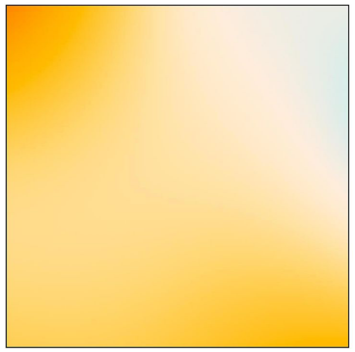
Gravitational Waves

New window into physics.

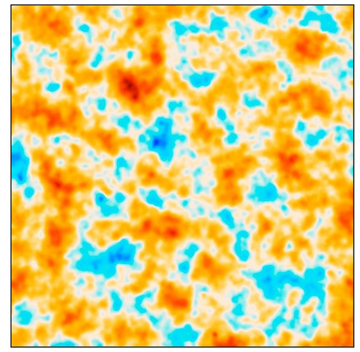


LIGO marks only the begining.

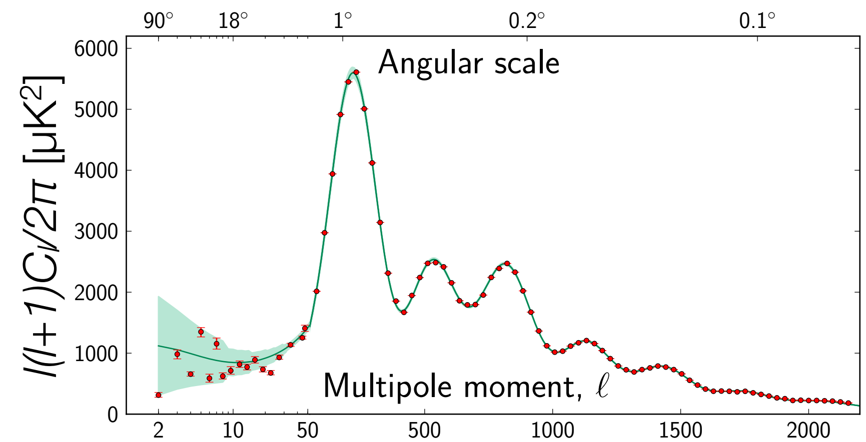
Theoretical Precision

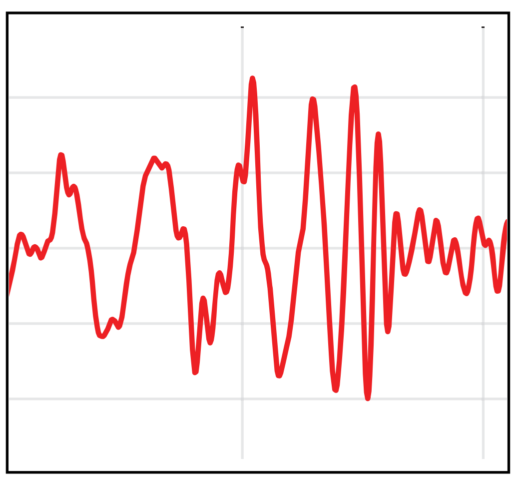


COBE

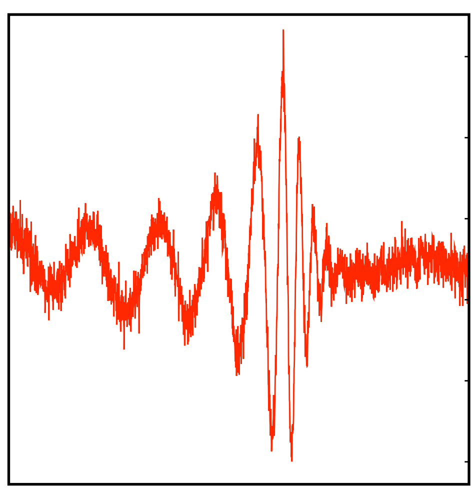


Planck

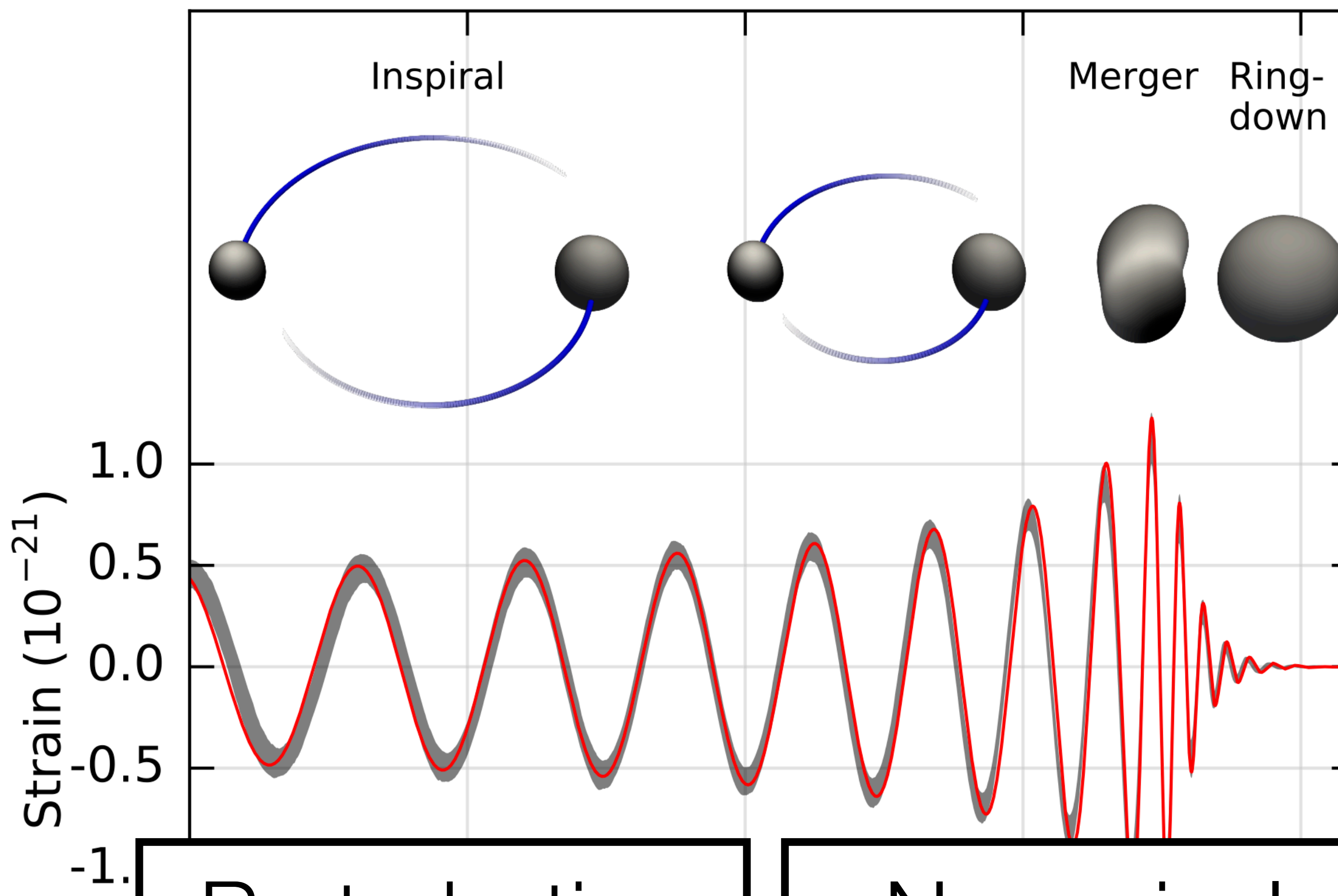




LIGO



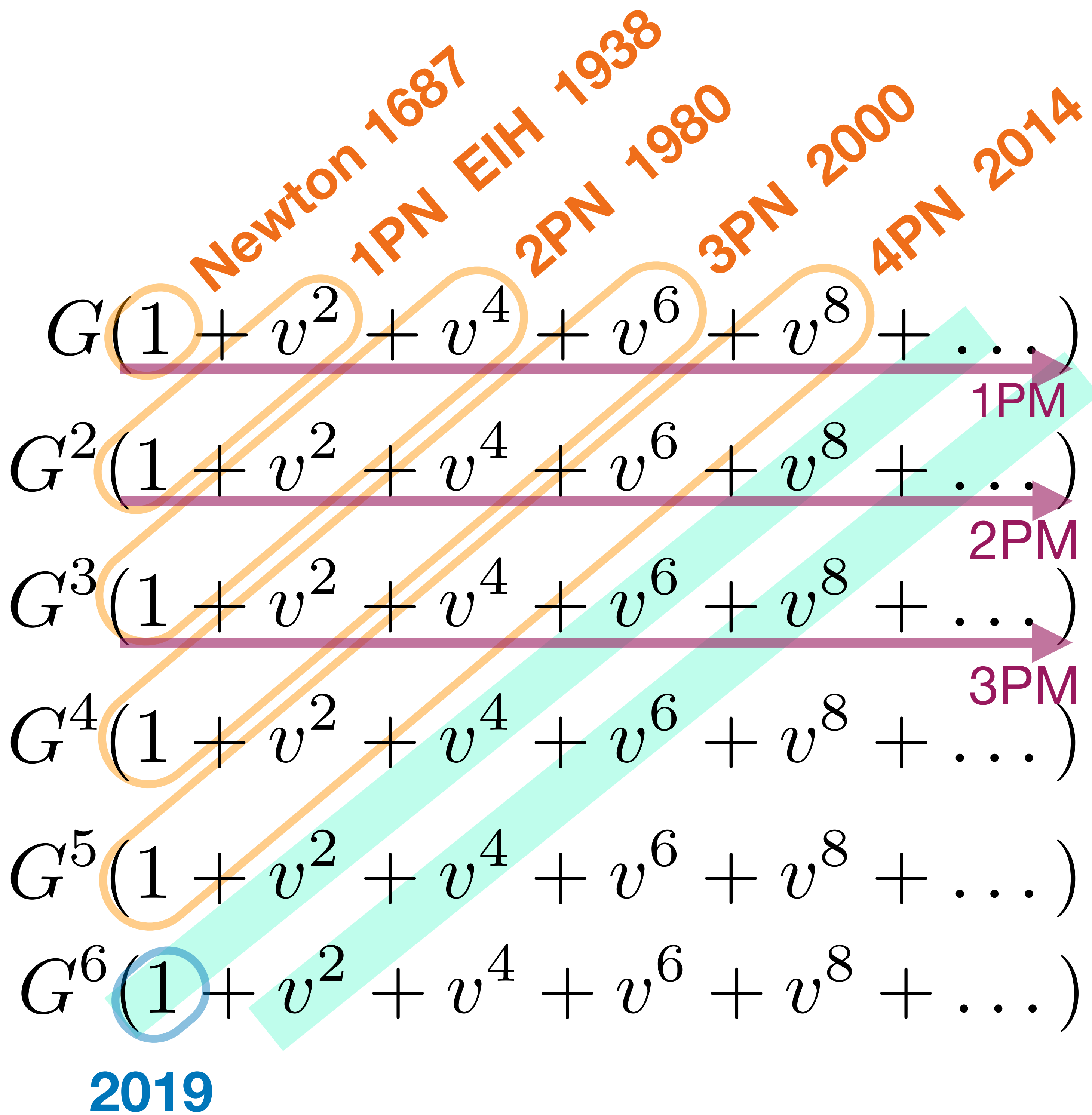
LISA



Perturbation Theory

Numerical Relativity

Perturbation Theory



post-Newtonian
$GM/r \sim v^2 \ll 1$

Blanchet, Damour, Mastrolia, …

post-Minkowskian
$GM/r \ll 1$

Ledvinka, Schäfer, Bicak 2008

Westpfahl, Goller 1979, Damour 2016
Cheung, Rothstein, **Solon** 2018

Bern, Cheung, Roiban, Shen, **Solon**, Zeng 2019

5PN: biased parameter estimates, tidal effects

6PN+: LISA, ET

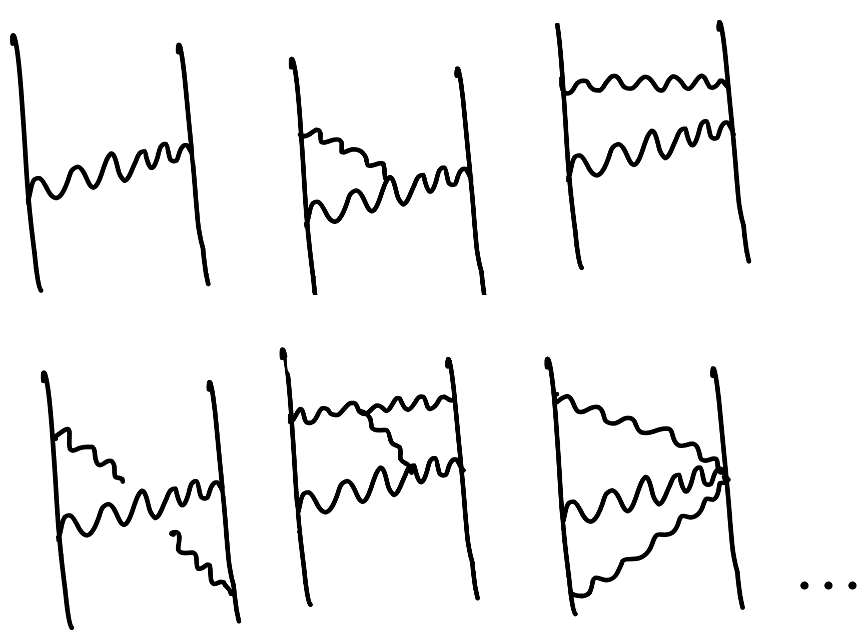
Scalability is key.

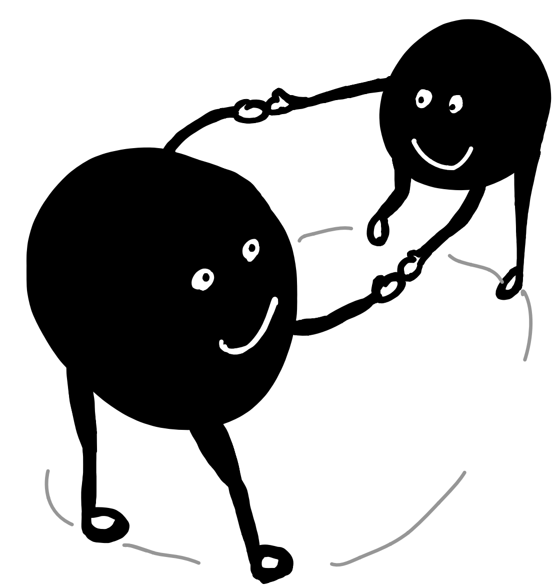
Effective Field Theory

Scattering Amplitudes

Quantum Field Theory

Binary Inspiral





New Result in Relativity

Bern, Cheung, Roiban, Shen, **Solon**, Zeng 2019

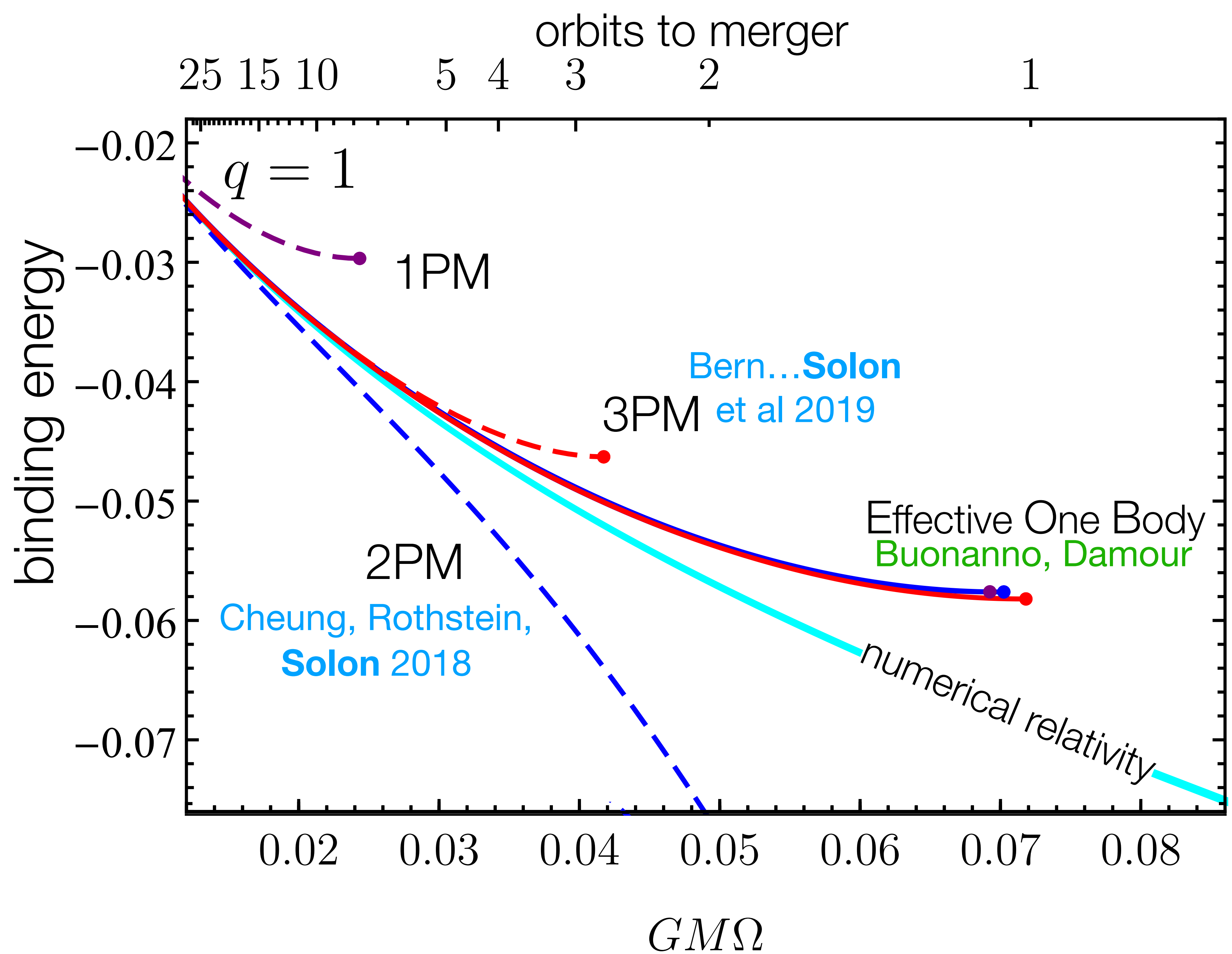
$$H^{3\mathrm{PM}}(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2+m_1^2} + \sqrt{\boldsymbol{p}^2+m_2^2} + \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$m = m_1+m_2\,,\ \nu = \frac{m_1 m_2}{m^2}\,,\ E = E_1+E_2\,,\ \xi = \frac{E_1E_2}{E^2}\,,\ \gamma = \frac{E}{m}\,,\ \sigma = \frac{p_1\cdot p_2}{m_1 m_2}$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right) \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right]$$

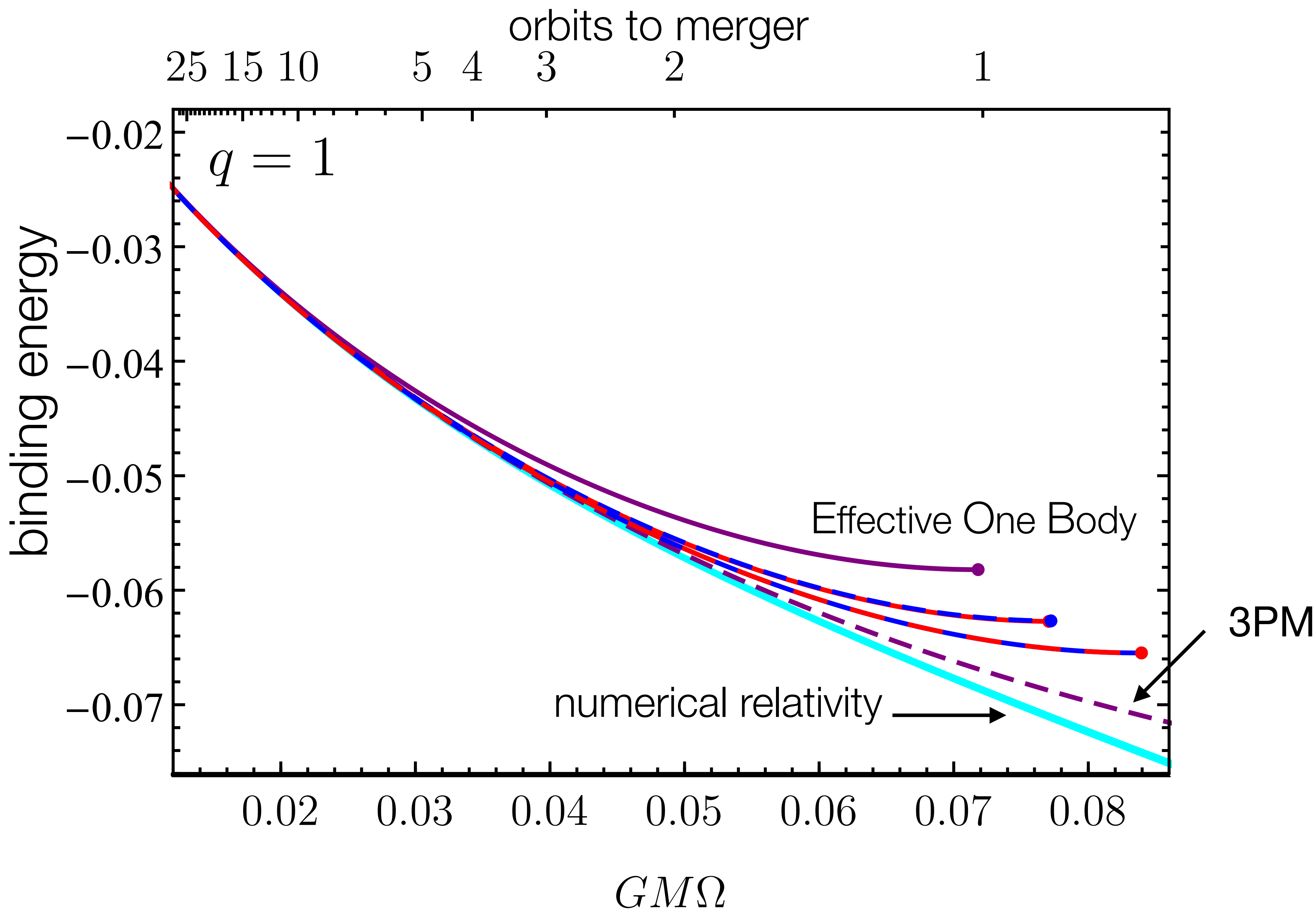
$$\begin{aligned} c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\Bigg[&\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \\ &- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} \\ &+ \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\Bigg], \end{aligned}$$

New Result in Relativity



Antonelli, Buonanno, Steinhoff, van de Meent, Vines 1901.07102

New Result in Relativity



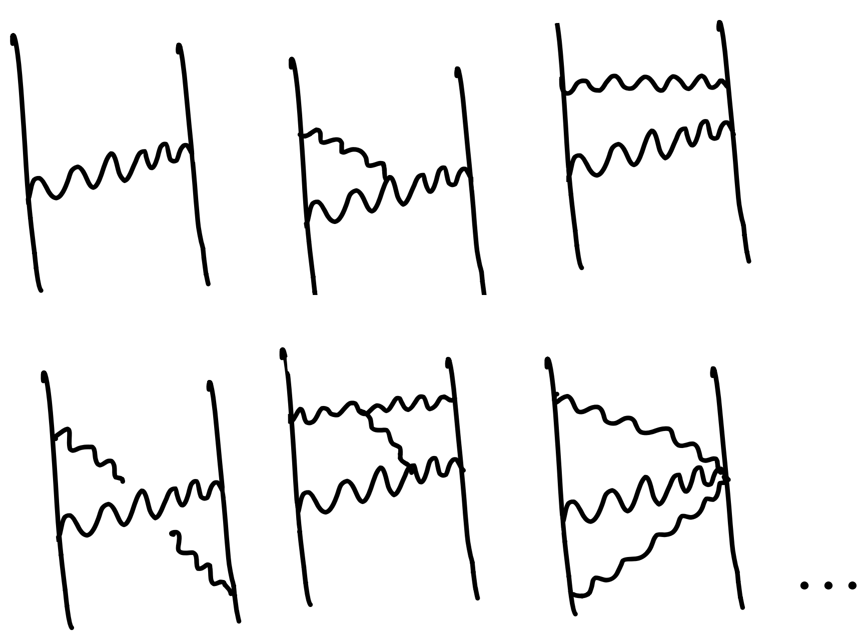
Antonelli, Buonanno, Steinhoff, van de Meent, Vines 1901.07102

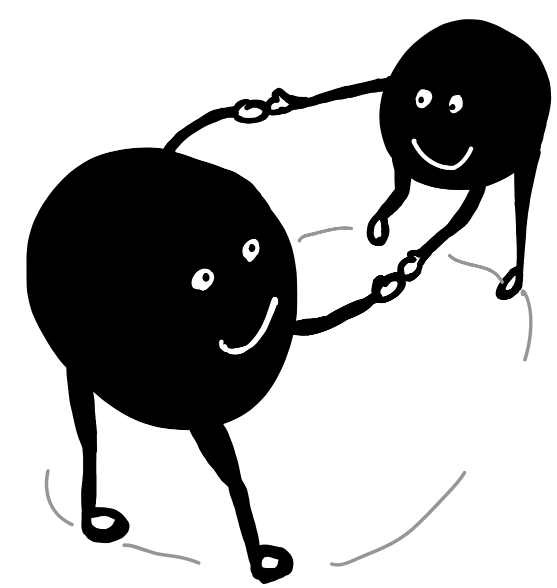
Effective Field Theory

Scattering Amplitudes

Quantum Field Theory

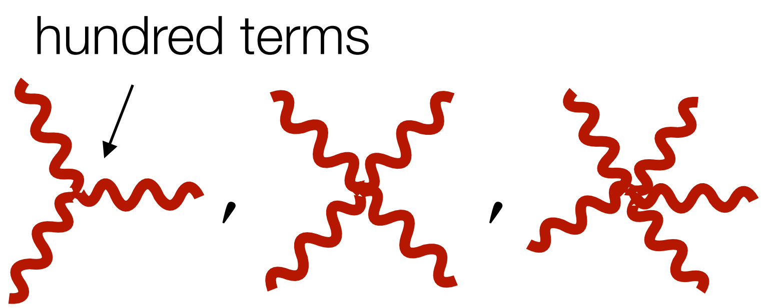
Binary Inspiral





Scattering Amplitudes

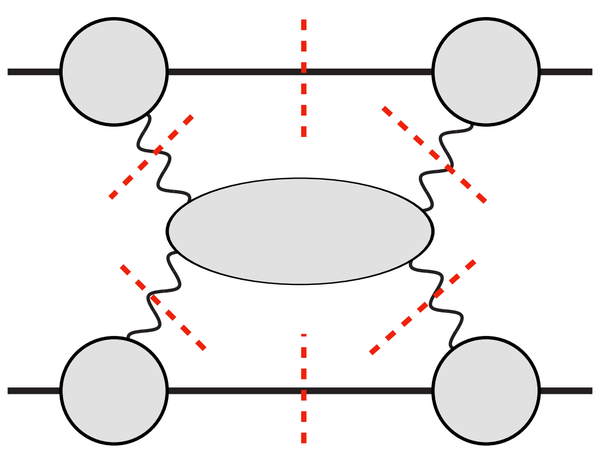
Feynman diagrams won't scale

$\mathcal{L}_{\mathrm{GR}} \sim$ 

...

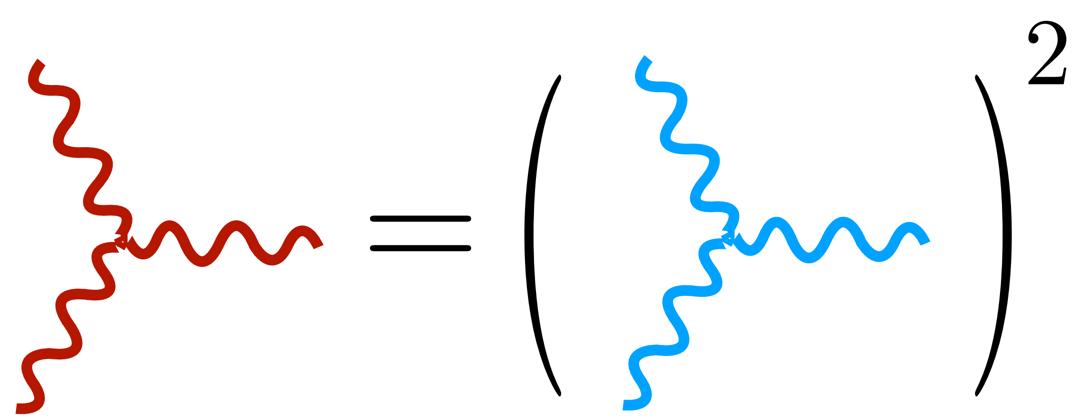
two-to-two graviton scattering has 10^{20} terms at three loops

On-shell methods are powerful.



Generalized Unitarity

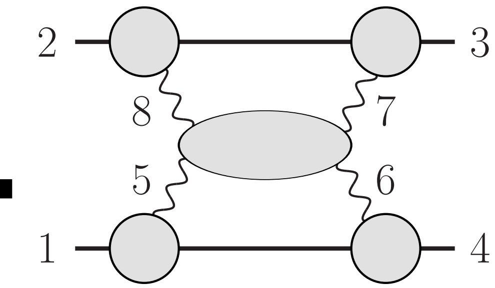
Bern, Dixon, Dunbar, Kosower



Double Copy

Kawai, Lewellen, Tye
Bern, Carrasco, Johansson

e.g. H Cut in D=4



Product of three GR four-point amplitudes, obtained from YM amplitudes

$$C^{2,2} = \sum_{\text{states}} M_4(2^s, -8, 7, 3^s) M_4(-5, 6, -7, 8) M_4(1^s, 5, -6, 4^s)$$

$$s_{ij} = (p_i + p_j)^2$$

$$M_4(1,2,3,4) = -i s_{12} A_4(1,2,3,4)\, A_4(1,2,4,3)$$

$$t_{ij} = 2 p_i \cdot p_j$$

$$A_4(1^s, 2^+, 3^+, 4^s) = i\, \frac{m_1^2\, [2\,3]}{\langle 2\,3\rangle\, t_{12}} \qquad A_4(1^-, 2^-, 3^+, 4^+) = i\, \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,1\rangle}$$

$$A_4(1^s, 2^+, 3^-, 4^s) = i\, \frac{\langle 3|\,1\,|2]^2}{t_{23}\, t_{12}} \qquad A_4(1^-, 2^+, 3^-, 4^+) = i\, \frac{\langle 1\,3\rangle^4}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,1\rangle}$$

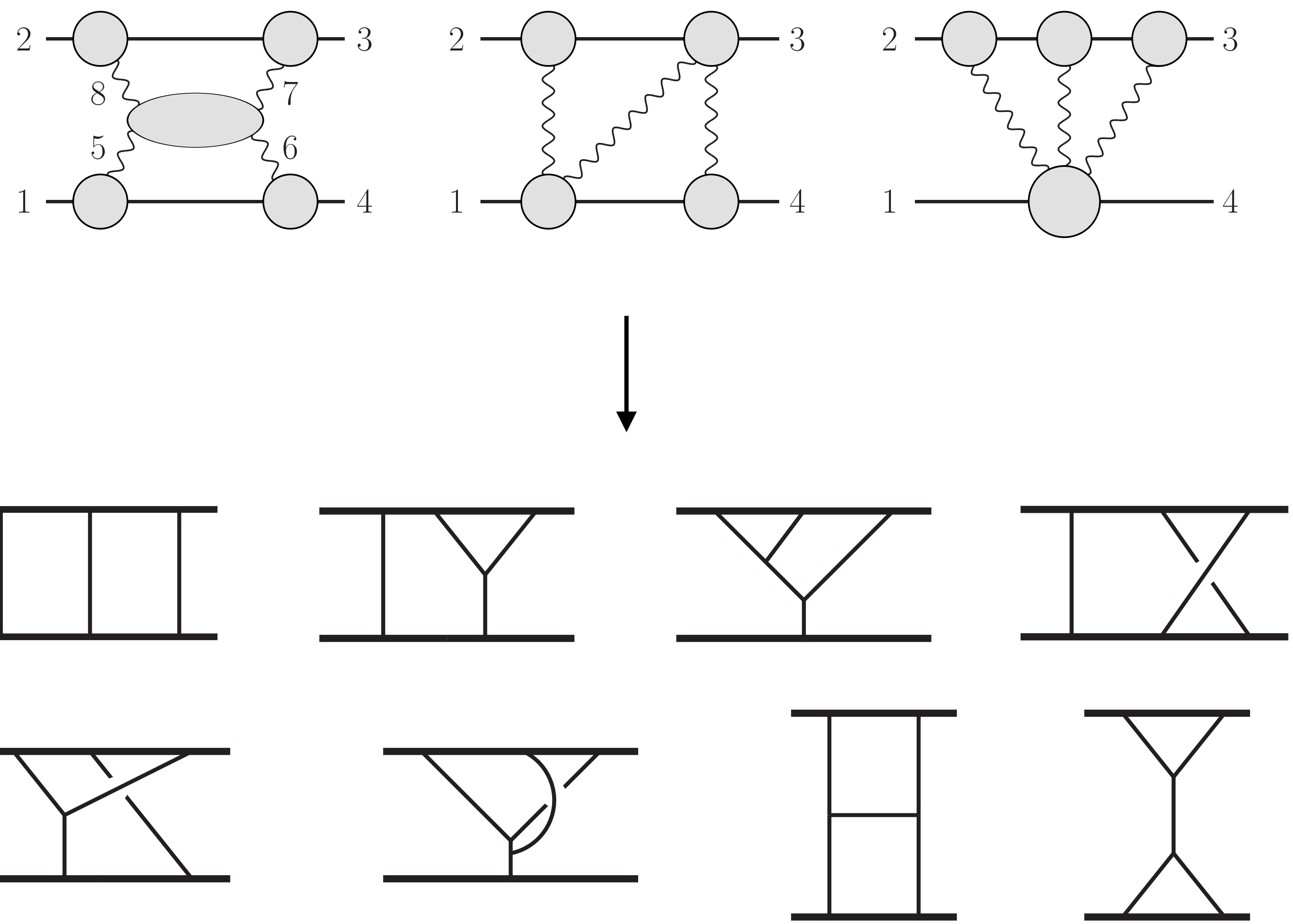
(gluon)^2 = graviton ~~+ dilaton + axion~~ by correlating gluon helicities in copies

$$C^{\text{H-cut}} = 2i \left[\frac{1}{(p_5 - p_8)^2} + \frac{1}{(p_5 + p_7)^2} \right] \left[s_{23}^2 m_1^4 m_2^4 + \frac{1}{s_{23}^6} \sum_{i=1,2} \left(\mathcal{E}_i^4 + \mathcal{O}_i^4 + 6 \mathcal{O}_i^2 \mathcal{E}_i^2 \right) \right]$$

$$\mathcal{E}_1^2 = \frac{1}{4} s_{23}^2 (t_{18} t_{25} - t_{12} t_{58})^2, \qquad \mathcal{O}_1^2 = \mathcal{E}_1^2 - m_1^2 m_2^2 s_{23}^2 t_{58}^2,$$

$$\mathcal{E}_2^2 = \frac{1}{4} s_{23}^2 (t_{17} t_{25} - t_{12} t_{57} - s_{23}(t_{17} + t_{57}))^2, \qquad \mathcal{O}_2^2 = \mathcal{E}_2^2 - m_1^2 m_2^2 s_{23}^2 t_{57}^2.$$

Other Cuts



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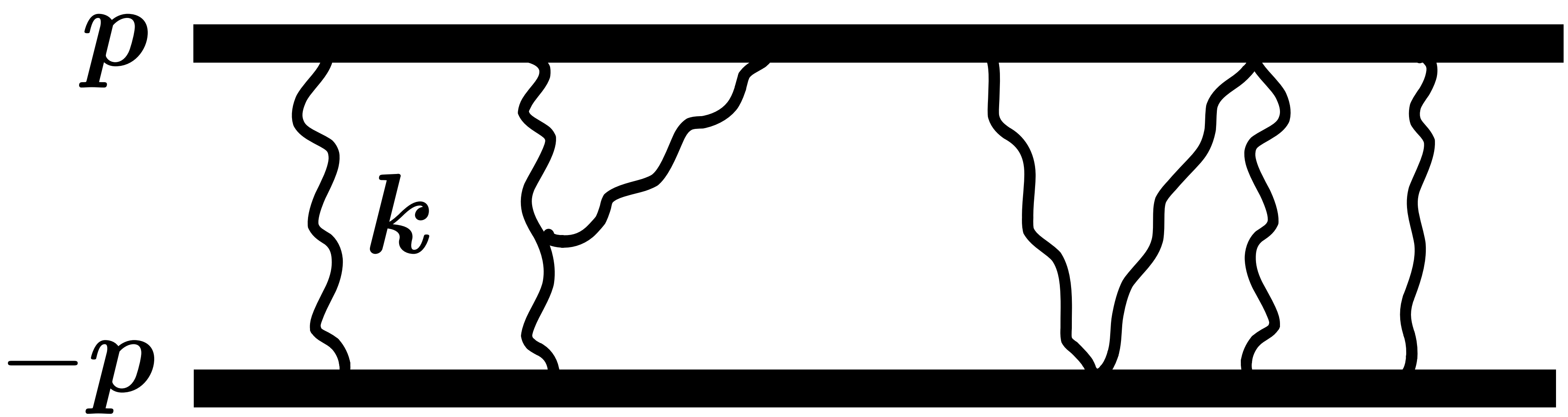
Effective Field Theory

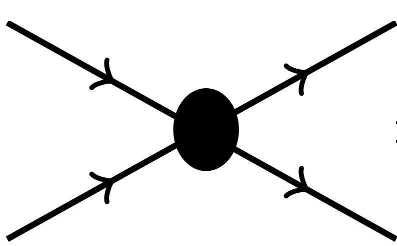
$$k_0 \ll |\boldsymbol{k}| \quad \ll \quad |\boldsymbol{p}| \ll m_i$$

potential   classical   nonrelativistic



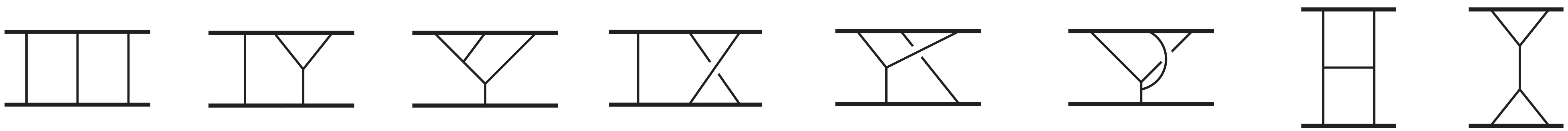
1. New integration strategy for full theory

2. Simple theory of scattering with an ansatz classical potential



$$= -iV(\boldsymbol{p}, \boldsymbol{q}) \qquad V(\boldsymbol{p}, \boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2) \left(\frac{G}{|\boldsymbol{r}|} \right)^i$$

Integration Strategy



$+(2 \leftrightarrow 3) + (1 \leftrightarrow 2)$

$$\mathcal{I} = \left[\prod_{i=1}^{n_M} \frac{1}{\varepsilon_i^2 - \boldsymbol{k}_i^2 - m_i^2}\right]\left[\prod_{j=1}^{n_G} \frac{1}{\omega_j^2 - \boldsymbol{\ell}_j^2}\right]\mathcal{N}$$

$$\ell = (\omega\,, \boldsymbol{\ell}) \qquad \mathrm{d}\ell = \mathrm{d}\boldsymbol{\ell}\,\mathrm{d}\omega \qquad \omega \sim \frac{|\boldsymbol{p}||\boldsymbol{q}|}{m} \ll |\boldsymbol{\ell}| \sim |\boldsymbol{q}|$$

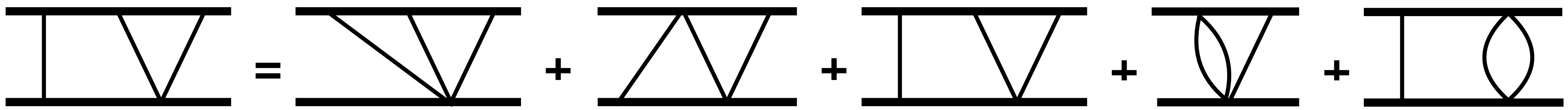
graviton momenta

① matter pole form

$$\mathcal{I} = \left[\prod_{i=1}^{n_M} \frac{1}{\varepsilon_i - \sqrt{\boldsymbol{k}_i^2 - m_i^2}}\right]\widetilde{\mathcal{N}} \qquad \widetilde{\mathcal{N}} = \left[\prod_{i=1}^{n_M} \frac{1}{\varepsilon_i + \sqrt{\boldsymbol{k}_i^2 + m_i^2}}\right]\left[\prod_{j=1}^{n_G} \frac{1}{\omega_j^2 - \boldsymbol{\ell}_j^2}\right]\mathcal{N}$$

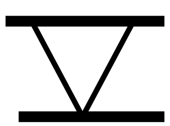
② energy integral reduction

$$\mathcal{I} = \left[\prod_{i=1}^{n_M} \frac{1}{\varepsilon_i - \sqrt{\boldsymbol{k}_i^2 - m_i^2}}\right] \widetilde{\mathcal{N}}(\omega_i)$$

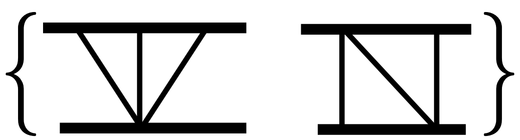


$$\int \mathrm{d}\omega\, \omega^n = 0\,,\ \ n \geq 0 \quad \Rightarrow \quad \prod_j \left[\varepsilon_j - \sqrt{\boldsymbol{k}_j^2 - m_j^2}\right] = 0 \text{ in } \widetilde{\mathcal{N}}$$

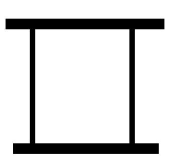
Akhoury, Saotome, Sterman



$$\omega - \omega_{P_1} \to 0 \qquad \int \frac{d\omega}{\omega - \omega_{P_1} + i\epsilon} = \frac{1}{2} \times (-2\pi i)$$



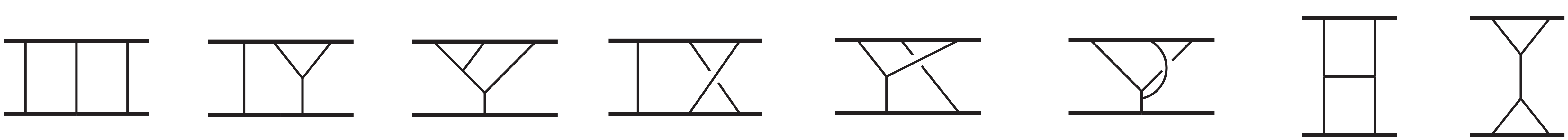
$$\omega_i - \omega_{P_i} \to 0 \qquad \left\{\frac{(-2\pi i)^2}{6}, -\frac{(-2\pi i)^2}{3}\right\}$$



$$(\omega - \omega_{P_1})(\omega - \omega_{P_2}) \to 0$$

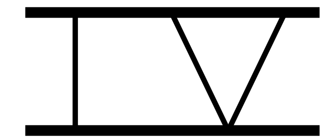
$$\widetilde{\mathcal{N}}(\omega) \to a\omega + b$$

$$\int \frac{d\omega}{(\omega - \omega_{P_1} + i\epsilon)(\omega - \omega_{P_2} - i\epsilon)} = \frac{2\pi i}{\omega_{P_2} - \omega_{P_1}} \sim \frac{1}{2\boldsymbol{p\ell} + \boldsymbol{\ell}^2} + \cdots$$



② 2’ residue method

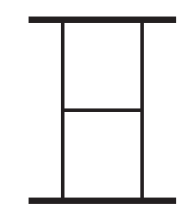
$$\int \frac{d\omega}{2\pi}\,\frac{d\omega'}{2\pi}\,\mathcal{I}(\omega,\omega') = \sum_{(i,j)} S_{ij} \operatorname*{Res}_{\omega_{ij},\omega'_{ij}} \mathcal{I}(\omega,\omega')$$



③ 3 spatial integration

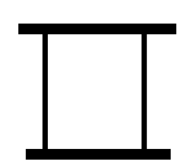
$$\mathcal{I} = \left[\prod_{i=1}^{n_M} \frac{1}{\varepsilon_i - \sqrt{\boldsymbol{k}_i^2 - m_i^2}}\right] \left[\prod_{i=1}^{n_M} \frac{1}{\varepsilon_i + \sqrt{\boldsymbol{k}_i^2 + m_i^2}}\right] \left[\prod_{j=1}^{n_G} \frac{1}{\omega_j^2 - \boldsymbol{\ell}_j^2}\right] \mathcal{N}$$

↓ potential, classical, nonrelativistic + IBP



$$\widetilde{\mathcal{I}} = \sum_{\alpha}\sum_{\beta}\sum_{\gamma} \frac{f^{(\alpha\beta\gamma)}(\boldsymbol{\ell})}{[\boldsymbol{\ell}^2]^\alpha[(\boldsymbol{\ell}+\boldsymbol{w})^2]^\beta[2\boldsymbol{z}\boldsymbol{\ell}+\boldsymbol{\ell}^2]^\gamma}$$

$\gamma = 0$ textbook: $\displaystyle\int \frac{d^{D-1}\boldsymbol{\ell}}{(2\pi)^{D-1}} \frac{\boldsymbol{\ell}^{\mu_1}\boldsymbol{\ell}^{\mu_2}\ldots\boldsymbol{\ell}^{\mu_n}}{[\boldsymbol{\ell}^2]^\alpha[(\boldsymbol{\ell}+\boldsymbol{w})^2]^\beta}$

$\gamma = 1$ IR artifacts:  $\displaystyle\sim \frac{1}{2\boldsymbol{p}\boldsymbol{\ell}+\boldsymbol{\ell}^2} + \cdots$

Full Theory Amplitude

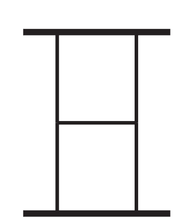
$$m = m_1 + m_2\,,\;\; \nu = \frac{m_1 m_2}{m^2}\,,\;\; E = E_1 + E_2\,,\;\; \xi = \frac{E_1 E_2}{E^2}\,,\;\; \gamma = \frac{E}{m}\,,\;\; \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$\mathcal{M}_1 = -\frac{4\pi G \nu^2 m^2}{\gamma^2 \xi \boldsymbol{q}^2}(1 - 2\sigma^2) \qquad \mathcal{M}_2 = -\frac{3\pi^2 G^2 \nu^2 m^3}{2\gamma^2 \xi |\boldsymbol{q}|}(1 - 5\sigma^2) + \int_{\boldsymbol{\ell}} \frac{32\pi^2 G^2 \nu^4 m^4 E (1 - 2\sigma^2)^2}{\gamma^4 \xi\, \boldsymbol{\ell}^2 (\boldsymbol{\ell} + \boldsymbol{q})^2 (2\boldsymbol{pl} + \boldsymbol{\ell}^2)}$$

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2 \xi}\Bigg[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma - 1}{2}}}{\sqrt{\sigma^2 - 1}}$$
$$- \frac{18\nu\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)}{(1 + \gamma)(1 + \sigma)}\Bigg] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4 \xi}\Big[3\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)F_1 - 32 m^2 \nu^2 \left(1 - 2\sigma^2\right)^3 F_2\Big]$$

$$F_1 = \int_{\boldsymbol{\ell}} \frac{1}{\boldsymbol{\ell}^2 |\boldsymbol{\ell} + \boldsymbol{q}| (2\boldsymbol{p\ell} + \boldsymbol{\ell}^2)} \qquad F_2 = \int_{\boldsymbol{\ell}_1, \boldsymbol{\ell}_2} \frac{1}{\boldsymbol{\ell}_1^2 (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2 (\boldsymbol{\ell}_2 + \boldsymbol{q})^2 (2\boldsymbol{p\ell}_1 + \boldsymbol{\ell}_1^2)(2\boldsymbol{p\ell}_2 + \boldsymbol{\ell}_2^2)}$$

- Note IR artifacts.
- Used dimreg to extract log[q]
- Real part only (conservative)
- Valid for $q \ll m$
- Resummed. Checked at 8PN and with Mellin-Barnes, IBP.



Effective Field Theory

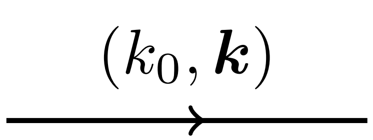
$$\mathcal{L} = \mathcal{L}_{\rm kin} - \int_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{k},\boldsymbol{k}')\, A^\dagger(\boldsymbol{k}')A(\boldsymbol{k})B^\dagger(-\boldsymbol{k}')B(-\boldsymbol{k})$$

$$V = \frac{c_1\kappa}{|\boldsymbol{q}|^2} + \frac{c_2\kappa^2}{|\boldsymbol{q}|} + c_3\kappa^2\log|\boldsymbol{q}| + \ldots$$

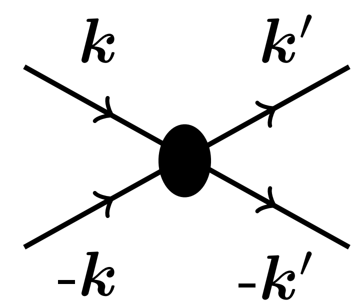
$$\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k}'$$

$$c_i = c_i\left[(\boldsymbol{k}^2 + \boldsymbol{k}'^2)/2\right]$$

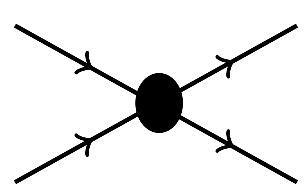
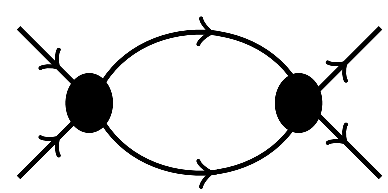
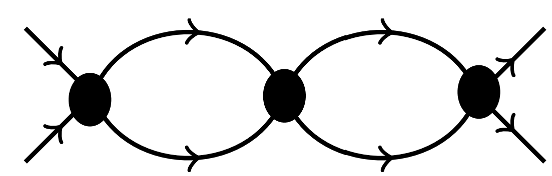
COM, real, gauge-dependent



$$= \frac{i}{k_0 - \sqrt{\boldsymbol{k}^2 + m_{A,B}^2} + i0}$$



$$= -iV(\boldsymbol{k},\boldsymbol{k}')\,,$$

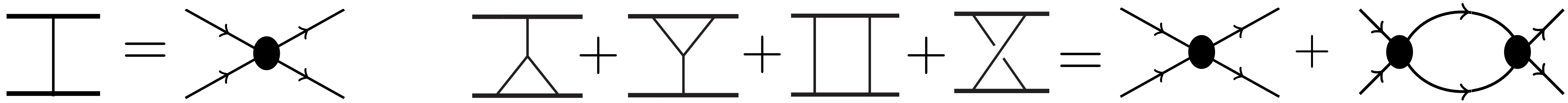
$$M^{(3)}_{\rm EFT} =$$  $+$  $+$ 

Effective Theory Amplitude

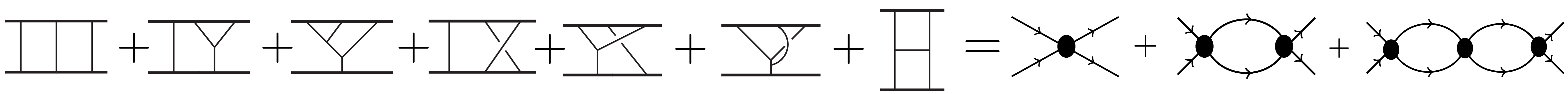
$$
\begin{aligned}
M_{\rm EFT}^{(1)} &= -\frac{\kappa c_1}{\boldsymbol{q}^2}\,, \\
M_{\rm EFT}^{(2)} &= -\frac{\kappa^2 c_2}{8|\boldsymbol{q}|} + \frac{\kappa^2}{16E\xi|\boldsymbol{q}|}\left[(1-3\xi)c_1^2 + 4\xi^2E^2c_1c_1'\right] + \int_{\boldsymbol{l}} \frac{2E\xi\kappa^2c_1^2}{\boldsymbol{l}^2|\boldsymbol{l}+\boldsymbol{q}|^2(\boldsymbol{l}^2+2\boldsymbol{p}\boldsymbol{l})}\,, \\
M_{\rm EFT}^{(3)} &= \frac{\kappa^3 c_3\log|\boldsymbol{q}|}{16\pi^2} + \frac{\kappa^3\log|\boldsymbol{q}|}{32\pi^2E^2\xi}\Big[(1-4\xi)c_1^3 - 8\xi^3E^4c_1{c_1'}^2 - 4\xi^3E^4c_1^2c_1'' + 4\xi^2E^3c_2c_1' \\
&+ \xi^2E^3c_1c_2' - 2(3-9\xi)\xi E^2c_1^2c_1' - 6\xi Ec_1c_2 + 2Ec_1c_2\Big] \\
&+ \int_{\boldsymbol{l}_1,\boldsymbol{l}_2} \frac{4T^2\xi^2\kappa^3c_1^3}{\boldsymbol{l}_1^2|\boldsymbol{l}_1+\boldsymbol{l}_2|^2|\boldsymbol{l}_2+\boldsymbol{q}|^2(\boldsymbol{l}_1^2+2\boldsymbol{p}\boldsymbol{l}_1)(\boldsymbol{l}_2^2+2\boldsymbol{p}\boldsymbol{l}_2)} + \int_{\boldsymbol{l}} \frac{2\kappa^3c_1^2\left[(1-3\xi)c_1+4\xi^2E^2c_1'\right]}{\boldsymbol{l}^2|\boldsymbol{l}+\boldsymbol{q}|(\boldsymbol{l}^2+2\boldsymbol{p}\boldsymbol{l})}
\end{aligned}
$$

* Note IR artifacts and subtractions.

Matching



$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1-2\sigma^2\right) \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right]$$



$$c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\left[\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\sinh^{-1}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\right],$$

New Result in Relativity

$$H^{\text{3PM}}(\boldsymbol{p}, \boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + \sum_{i=1}^{3} c_i(\boldsymbol{p}^2) \left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$m = m_1 + m_2\,,\;\; \nu = \frac{m_1 m_2}{m^2}\,,\;\; E = E_1 + E_2\,,\;\; \xi = \frac{E_1 E_2}{E^2}\,,\;\; \gamma = \frac{E}{m}\,,\;\; \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right) \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1 - 5\sigma^2\right) - \frac{4\nu\sigma\left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2 (1-\xi)\left(1 - 2\sigma^2\right)^2}{2\gamma^3 \xi^2}\right]$$

$$\begin{aligned} c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\Bigg[&\frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\sinh^{-1}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \\ &- \frac{3\nu\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1 - 2\sigma^2\right)}{4\gamma^3\xi^2} \\ &+ \frac{2\nu^3 (3 - 4\xi)\sigma\left(1 - 2\sigma^2\right)^2}{\gamma^4 \xi^3} + \frac{\nu^4 (1 - 2\xi)\left(1 - 2\sigma^2\right)^3}{2\gamma^6 \xi^4}\Bigg], \end{aligned}$$

Checks

Potentials e.g. from PN and NRGR are in different gauges:

$$V \supset \boldsymbol{p}^2 - \boldsymbol{p}'^2 \sim \boldsymbol{p}\cdot\boldsymbol{q} \sim \boldsymbol{p}\cdot\boldsymbol{r}$$

Compare to 4PN

Damour, Jaranowski, Schäfer 2014
Bernard, Blanchet, Boh, Faye, Marsat 2015
Jaranowski, Schäfer 2015

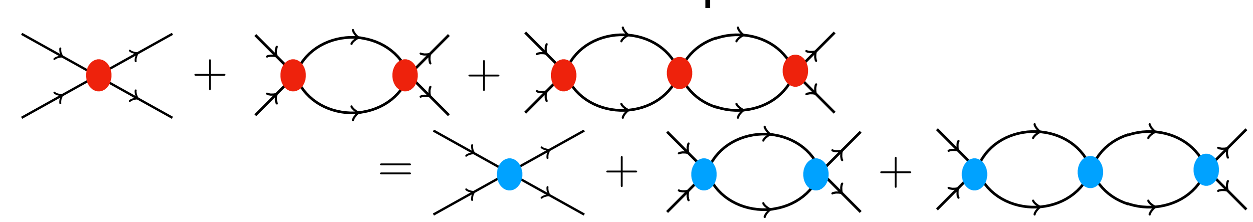
$$\begin{array}{l}
G(1 + v^2 + v^4 + v^6 + v^8 + \dots) \\
G^2(1 + v^2 + v^4 + v^6 + v^8 + \dots) \\
G^3(1 + v^2 + v^4 + v^6 + v^8 + \dots) \\
G^4(1 + v^2 + v^4 + v^6 + v^8 + \dots) \\
G^5(1 + v^2 + v^4 + v^6 + v^8 + \dots)
\end{array}$$

Construct diffeo to map Hamiltonians

$$(\boldsymbol{r},\boldsymbol{p}) \to (\boldsymbol{R},\boldsymbol{P}) = (A\,\boldsymbol{r} + B\,\boldsymbol{p}, C\,\boldsymbol{p} + D\,\boldsymbol{r})$$

$$\{\boldsymbol{r},\boldsymbol{p}\} = \{\boldsymbol{R},\boldsymbol{P}\} = \mathbf{1}$$

Compute on-shell amplitudes from different potentials



Checks

Compute physical quantities

- 2PN energy of circular orbit
- 4PN scattering angle Bini, Damour

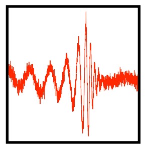
$$\chi = -\frac{m\gamma\xi\widetilde{M}_1}{2\pi L|\boldsymbol{p}|} - \frac{m\gamma\xi\widetilde{M}_2}{2\pi L^2} + \frac{2m\gamma\xi|\boldsymbol{p}|\widetilde{M}_3}{\pi L^3} - \frac{m^2\gamma^2\xi^2\widetilde{M}_1\widetilde{M}_2}{2\pi^3 L^3|\boldsymbol{p}|} + \frac{m^3\gamma^3\xi^3\widetilde{M}_1^3}{96\pi^3 L^3|\boldsymbol{p}|^3},$$

Compare to Schwarzchild in probe limit $m_1 \ll m_2$

- all orders in velocity

$$V_{\rm S} = \left(1 - \frac{Gm}{2r}\right)\left(1 + \frac{Gm}{2r}\right)^{-1}\sqrt{1 + \left(1 + \frac{Gm}{2r}\right)^{-4}\boldsymbol{p}^2} - 1$$

Conclusions



2020 2025 2030

aLIGO/Virgo KAGRA IndIGO TianQin DECIGO Einstein LISA

There's lots to do.

$$
\begin{aligned}
&G(1 + v^2 + v^4 + v^6 + v^8 + \ldots)\\
&G^2(1 + v^2 + v^4 + v^6 + v^8 + \ldots)\\
&G^3(1 + v^2 + v^4 + v^6 + v^8 + \ldots)\\
&G^4(1 + v^2 + v^4 + v^6 + v^8 + \ldots)\\
&G^5(1 + v^2 + v^4 + v^6 + v^8 + \ldots)\\
&G^6(1 + v^2 + v^4 + v^6 + v^8 + \ldots)
\end{aligned}
$$

* 5PN (bias), 6PN (LISA)
* radiation, spin, tidal
* further develop tools

Quantum field theory is useful for GW astrophysics.

Binary Black Holes and Gluon Scattering Amplitudes

Mikhail P. Solon

Caltech

based on work with

Clifford Cheung, Ira Rothstein (PRL)

Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mao Zeng (PRL)

Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mao Zeng (long paper)