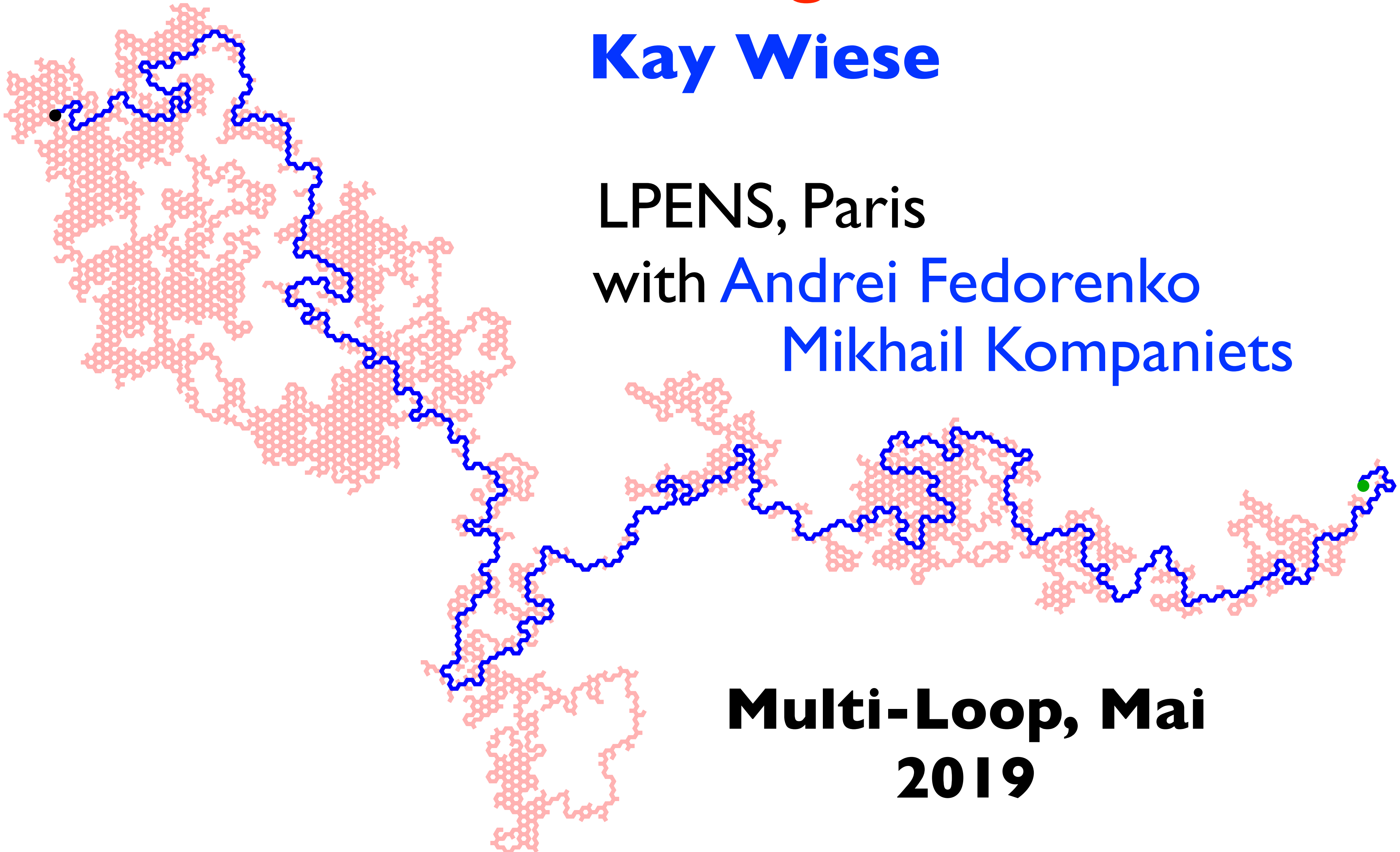
Field Theories for Loop-Erased Random Walks + Log CFTs



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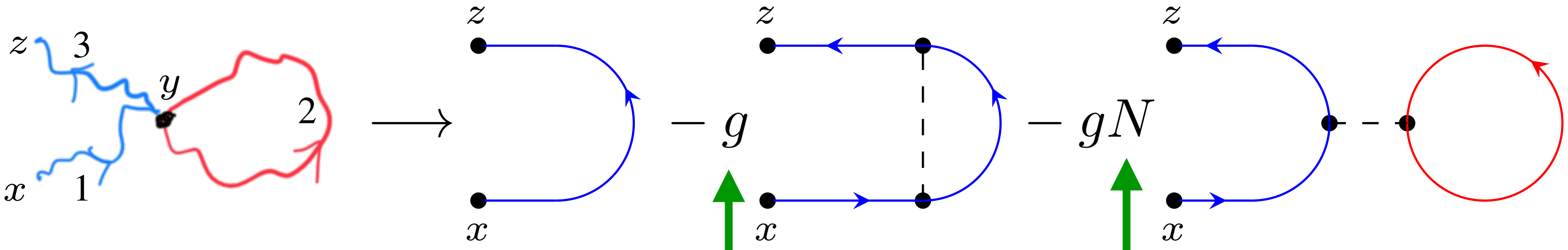
Multi-Loop, Mai 2019

http://www.phys.ens.fr/~wiese/

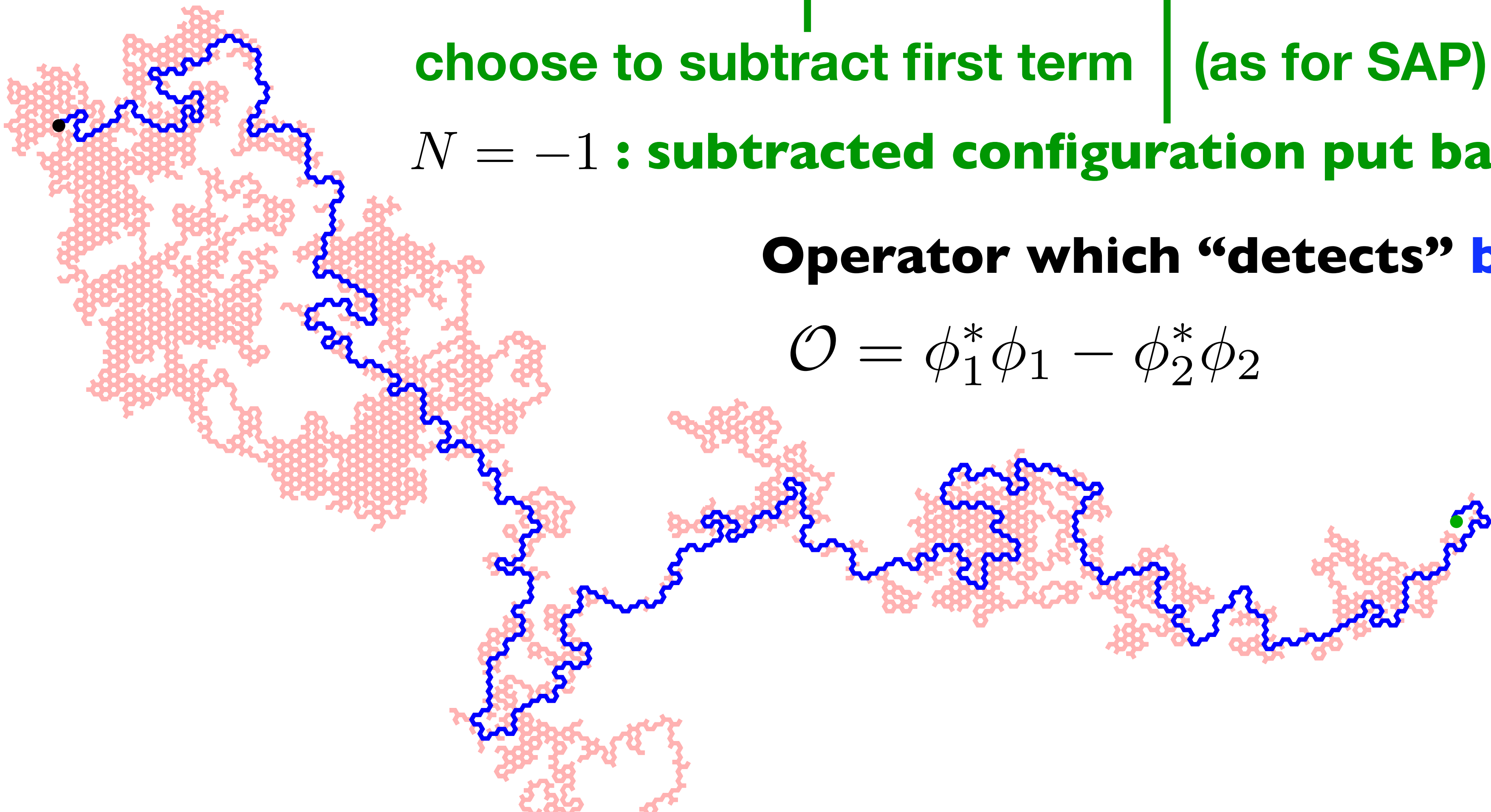
Loop-erased random walks = complex ϕ^4-theory at $N = -1$.

Random walk with one intersection



choose to subtract first term **(as for SAP)**

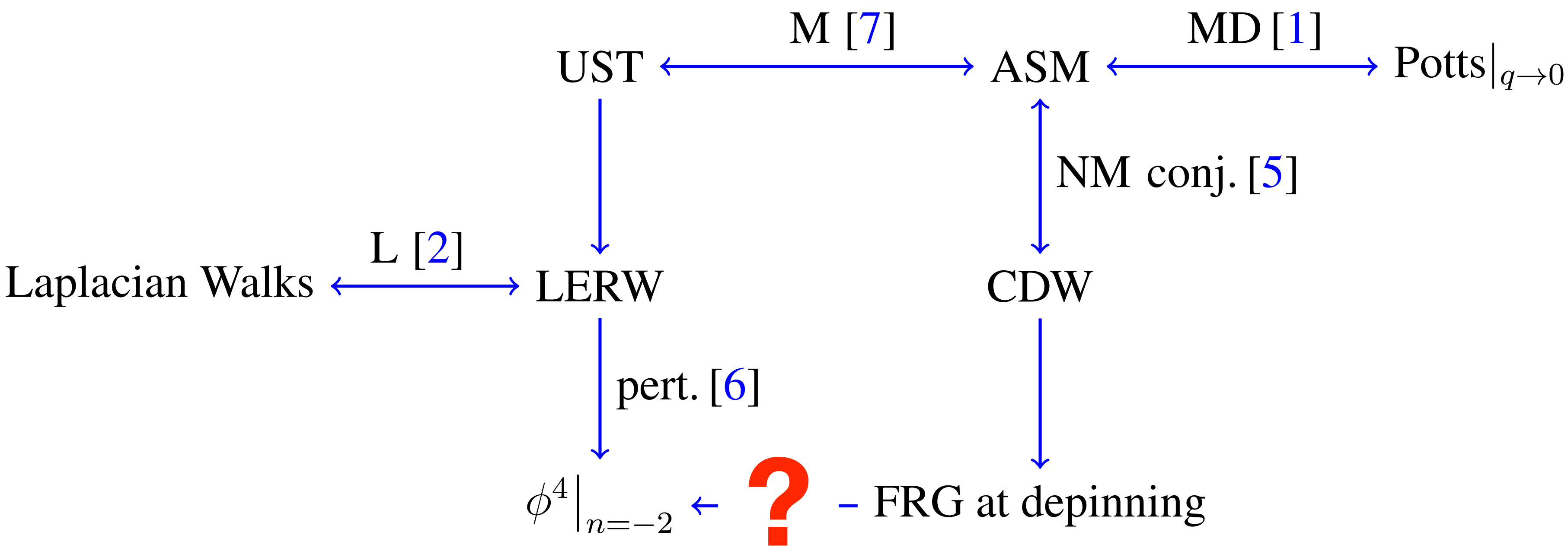
$N = -1$ **: subtracted configuration put back in red**



Operator which "detects" blue :

$$\mathcal{O} = \phi_1^* \phi_1 - \phi_2^* \phi_2$$

Relations



[1] S.N. Majumdar and D. Dhar, *Equivalence between the Abelian sandpile model and the $q \to 0$ limit of the Potts-model*, Physica A **185** (1992) 129–145.

[2] Gregory F. Lawler, *The Laplacian-b random walk and the Schramm-Loewner evolution*, Illinois J. Math. **50** (2006) 701–746.

[3] L. Niemeyer, L. Pietronero and H. J. Wiesmann, *Fractal dimension of dielectric breakdown*, Phys. Rev. Lett. **52** (1984) 1033–1036.

[4] F.Y. Wu, *Percolation and the potts model*, J. Stat. Phys **18** (1978) 115–123.

[5] O. Narayan and A.A. Middleton, *Avalanches and the renormalization-group for pinned charge-density waves*, Phys. Rev. B **49** (1994) 244–256.

[6] K.J. Wiese and A.A. Fedorenko, *Field theories for loop-erased random walks*, (2018), arXiv:1802.08830.

[7] S.N. Majumdar, *Exact fractal dimension of the loop-erased self-avoiding walk in two dimensions*, Phys. Rev. Lett. **68** (1992) 2329–2331.

Field Theory for Charge Density Waves (CDW)

- semi-conductor devices may have an instability for a periodic modulation of the charge density $\longrightarrow$ CDW

$$\mathcal{H}[u] := \int_x \frac{1}{2}[\nabla u(x)]^2 + \frac{m^2}{2}[u(x) - w]^2 + V\big(x, (u(x)\big)$$

↑ **disorder**

- disorder force correlator

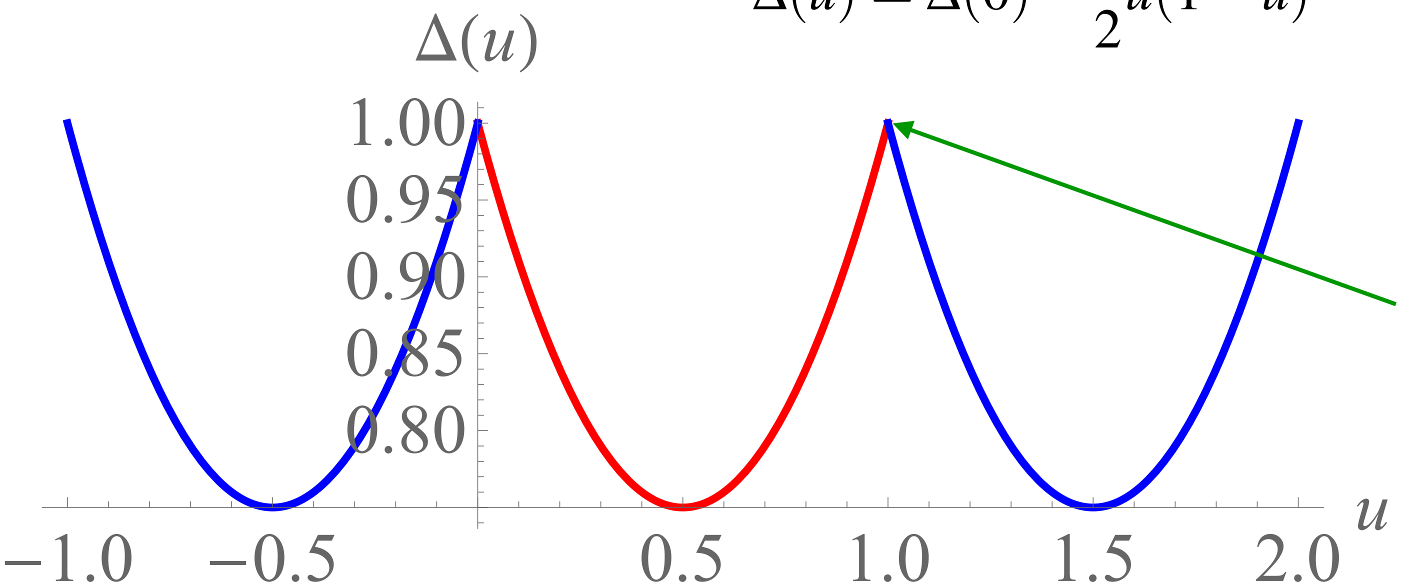
$$\overline{\partial_u V(x,u)\partial_{u'} V(x',u')} = \delta^d(x-x')\Delta(u-u')$$

- renormalizes under RG

$$-\frac{m\mathrm{d}}{\mathrm{d}m}\Delta(u) = (\varepsilon - 2\zeta)\Delta(u) + \zeta u \Delta'(u) - \partial_u^2 \left[\frac{1}{2}\Delta(u)^2 - \Delta(u)\Delta(0)\right]$$

CDW: $\zeta = 0$ and periodic fixed point $\Delta(u)$, which is piecewise

$$\Delta(u) = \Delta(0) - \frac{g}{2}u(1-u)$$



$$|\Delta'(0^+)| = m^4 L^{-d} \frac{\langle S^2 \rangle}{2\langle S \rangle}$$

↑ **avalanche moments**

Charge Density Waves (CDW) → ϕ^4-theory at *N*=-1

Action at depinning

$$\mathcal{S}^{\rm CDW} = \int_{x,t} \tilde u(x,t)(\partial_t - \nabla^2 + m^2)u(x,t) - \frac{1}{2}\int_{x,t,t'} \tilde u(x,t)\tilde u(x,t')\Delta\big(u(x,t)-u(x,t')\big).$$

FRG fixed point function for CDWs at depinning

$$\Delta(u) = \Delta(0) - \frac{g}{2}u(1-u)$$

difference $\phi(x)$ between 2 copies

Keep only leading term ~ $g\,u^2/2$

$$\mathcal{S}^{\rm CDW}_{\rm simp} := \int_{x,t} \tilde u(x,t)(\partial_t - \nabla^2 + m^2)u(x,t) - \frac{g}{4}\int_{x,t,t'} \tilde u(x,t)\tilde u(x,t')\big[u(x,t)-u(x,t')\big]^2$$

Redo with Supersymmetry

$$\mathcal{S} = \int_x \tilde\phi(x)(-\nabla^2+m^2)\phi(x) + \tilde u(x)(-\nabla^2+m^2)u(x) + \sum_{a=1}^{2}\bar\psi_a(x)(-\nabla^2+m^2)\psi_a(x)$$
$$+\frac{g}{2}\tilde u(x)\phi(x)\Big[\bar\psi_2(x)\psi_2(x) - \bar\psi_1(x)\psi_1(x)\Big] - \frac{g}{8}\tilde u(x)^2\phi(x)^2$$
$$+\frac{g}{2}\Big[\tilde\phi(x)\phi(x) + \bar\psi_1(x)\psi_1(x) + \bar\psi_2(x)\psi_2(x)\Big]^2 .$$

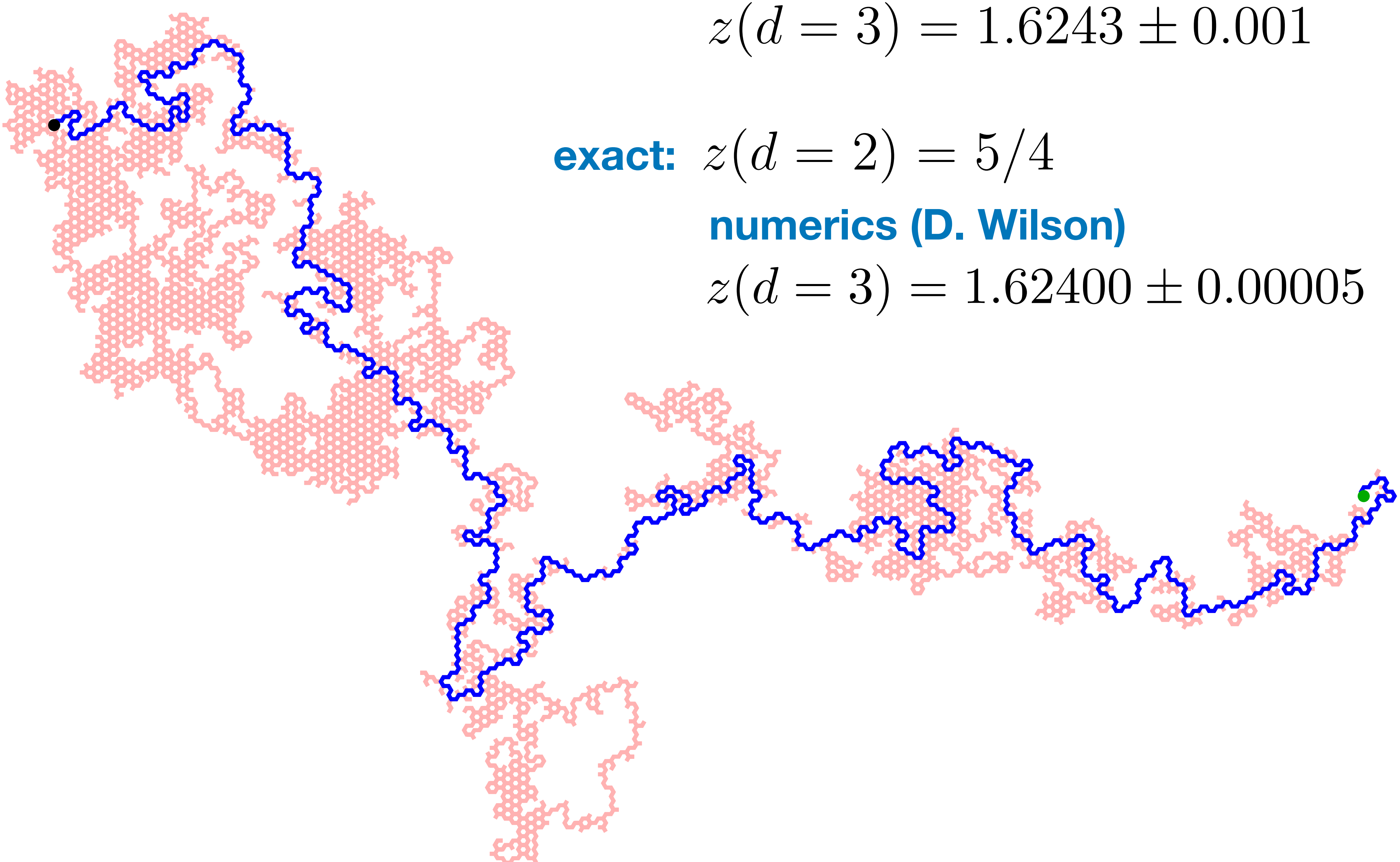
← decouple

multiplet out of 2 fermions + 1 boson = -1 boson

Numerical values for fractal dimension z

6 loops: $z(d=2) = 1.244 \pm 0.01,$

$z(d=3) = 1.6243 \pm 0.001$



exact: $z(d=2) = 5/4$

numerics (D. Wilson)

$z(d=3) = 1.62400 \pm 0.00005$

The curve detecting operator

$$\phi_i \phi_j^* - \delta_{ij} \frac{1}{n} \sum_k \phi_k \phi_k^*$$

and its applications

$n = -2$ loop erased random walks (new)

$n = 0$ self-avoiding polymers

$n = 1$ propagator line in the Ising model (to be proven)

Log-CFT for self-avoiding polymers

Define in the O(n) model

$$\mathcal{E}_i := \phi_i^2 \,, \qquad \mathcal{E} := \frac{1}{n}\sum_{i=1}^{n} \phi_i^2 \qquad\qquad \tilde{\mathcal{E}}_i := \phi_i^2 - \frac{1}{n}\sum_{j=1}^{n}\phi_j^2 \equiv \mathcal{E}_i - \mathcal{E}$$

They have dimension

$$x_{\mathcal{E}}(n) = \dim_\mu(\mathcal{E})$$
$$x_{\tilde{\mathcal{E}}}(n) = \dim_\mu(\tilde{\mathcal{E}})$$

Correlation functions

$$\langle \mathcal{E}(r)\mathcal{E}(0)\rangle = \frac{1}{n}\Big[\langle \mathcal{E}_1(r)\mathcal{E}_1(0)\rangle + (n-1)\langle \mathcal{E}_1(r)\mathcal{E}_2(0)\rangle\Big] \simeq \frac{A(n)}{n} r^{-2x_{\mathcal{E}}(n)}$$
$$\left\langle \tilde{\mathcal{E}}_i(r)\tilde{\mathcal{E}}_i(0)\right\rangle = \frac{n-1}{n}\Big[\langle \mathcal{E}_1(r)\mathcal{E}_1(0)\rangle - \langle \mathcal{E}_1(r)\mathcal{E}_2(0)\rangle\Big] \simeq \frac{n-1}{n}\tilde{A}(n) r^{-2x_{\tilde{\mathcal{E}}}(n)}$$

Since for $n \to 0$ both operators are identical, $A(0) = \tilde{A}(0)$. Define

$$\mathcal{C} := \lim_{n\to 0}[x_{\mathcal{E}}(n) - x_{\tilde{\mathcal{E}}}(n)]\mathcal{E} \equiv \lim_{n\to 0}[x_{\mathcal{E}}(n) - x_{\tilde{\mathcal{E}}}(n)]\tilde{\mathcal{E}}$$
$$\mathcal{D} := \lim_{n\to 0} \mathcal{E} - \tilde{\mathcal{E}}$$

$\mathcal{C}$ and $\mathcal{D}$ form a logarithmic pair

$$\mathcal{C} := \lim_{n\to 0}[x_{\mathcal{E}}(n) - x_{\tilde{\mathcal{E}}}(n)]\mathcal{E} \equiv \lim_{n\to 0}[x_{\mathcal{E}}(n) - x_{\tilde{\mathcal{E}}}(n)]\tilde{\mathcal{E}}$$

$$\mathcal{D} := \lim_{n\to 0} \mathcal{E} - \tilde{\mathcal{E}}$$

This implies

$$\langle \mathcal{D}(0)\mathcal{D}(r)\rangle = \lim_{n\to 0}\frac{1}{n}\left[A(n)r^{-2x_{\mathcal{E}}(n)} - \tilde{A}(n)r^{-2x_{\tilde{\mathcal{E}}}(n)}\right] = -\frac{-2\alpha\ln(r) + \text{const}}{r^{2x(0)}}$$

$$\langle \mathcal{C}(0)\mathcal{D}(r)\rangle = \lim_{n\to 0}[x_{\mathcal{E}}(n) - x_{\tilde{\mathcal{E}}}(n)]\left\langle \mathcal{E}(0)[\mathcal{E}(r) - \tilde{\mathcal{E}}(0)]\right\rangle = \frac{\alpha}{r^{2x(0)}}$$

$$\langle \mathcal{C}(0)\mathcal{C}(r)\rangle = \lim_{n\to 0}[x_{\mathcal{E}}(n) - x_{\tilde{\mathcal{E}}}(n)]^2\langle \mathcal{E}(0)\mathcal{E}(r)\rangle = 0\ .$$

$$\alpha = A(0)\Big(x'_{\mathcal{E}}(0) - x'_{\tilde{\mathcal{E}}}(0)\Big) \equiv \tilde{A}(0)\Big(x'_{\mathcal{E}}(0) - x'_{\tilde{\mathcal{E}}}(0)\Big)\ .$$

Action of the dilation operator

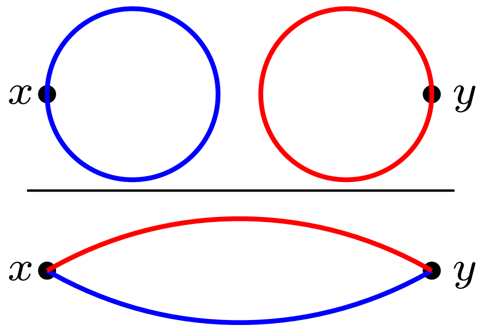
$$\mathbb{D}\circ\mathcal{E} = x_{\mathcal{E}}(n)\,\mathcal{E}$$

$$\mathbb{D}\circ\tilde{\mathcal{E}} = x_{\tilde{\mathcal{E}}}(n)\,\tilde{\mathcal{E}}$$

Jordan block under dilation

$$\mathbb{D}\circ\begin{pmatrix}\mathcal{C}\\ \mathcal{C}\end{pmatrix} = \begin{pmatrix}x & 0\\ 1 & x\end{pmatrix}\begin{pmatrix}\mathcal{C}\\ \mathcal{C}\end{pmatrix}$$

Physical Prediction for Polymer Correlation Function



$$= \text{const.} + 4\ln(|x-y|)\Big(x'_{\tilde{\mathcal{E}}}(0) - x'_{\mathcal{E}}(0)\Big)$$

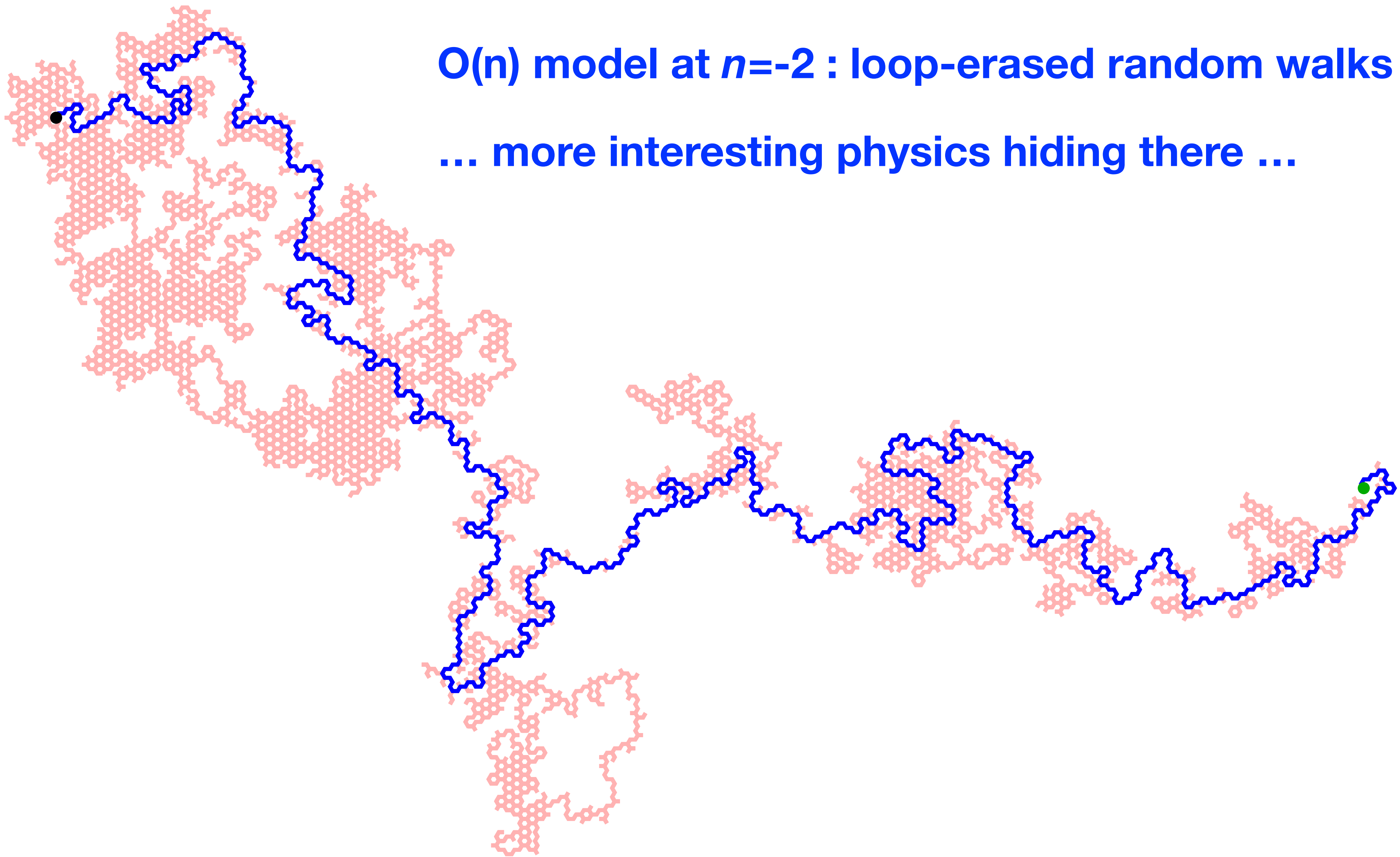
Preliminary result for amplitude

$$4\Big(x'_{\tilde{\mathcal{E}}}(0) - x'_{\mathcal{E}}(0)\Big) = \begin{cases} -0.63 \pm 0.02, & d=3 \\ -1.4 \pm 0.1, & d=2 \end{cases}$$

Prediction in *d*=2 from CFT

$$4\Big(x'_{\tilde{\mathcal{E}}}(0) - x'_{\mathcal{E}}(0)\Big) = -\frac{4}{\pi} = -1.27324\ldots$$

Conclusions



O(n) model at n=-2 : loop-erased random walks

… more interesting physics hiding there …